Math 309, Section 2 – Exam 3 Review

Exam 3 will be given in class on Monday, Nov. 19. The exam covers the material of Sections 4.3 and 5.1-5.6 of the textbook. Here is a list of topics to study:

1. Definitions. You should be able to give *precise* definitions of the following terms:

Linear Operator	Similar matrices	Dot product
Norm $\|\mathbf{v}\|$	Orthogonal vectors	Orthogonal subspaces
Unit vector	Component of $\mathbf{v}$ along $\mathbf{w}$	Projection of $\mathbf{v}$ onto $\mathbf{w}$
Deviation-from-mean vector	Correlation	Residue
Orthogonal complement	Distance between vectors in $\mathbb{R}^n$	Least Squares Solution
inner product	Cauchy-Schwartz Inequality	Triangle Inequality
orthogonal set	orthonormal set	orthogonal matrix.

2. Theorems. You should know and be able to apply the lemmas and theorems given in class and in Sections 4.3-5.6 of the textbook.

3. Calculations. You should know how to

- Check whether two matrices are similar.
- Given linear operator $L : V \to V$ and bases $A = \{\mathbf{v}_1, \dots \mathbf{v}_n\}$ and $B = \{\mathbf{w}_1, \dots \mathbf{w}_n\}$ of V, find the matrix of L in the A basis, in the B basis, and the transition matrix S relating them.
- Compute dot products, norms, the component of $\mathbf{v}$ along $\mathbf{w}$, and the projection $\mathrm{Proj}_{\mathbf{v}}(\mathbf{w})$.
- Check if subspaces of $\mathbb{R}^n$ are orthogonal. Given a subspace S, find a basis of $S^\perp$. Use formula $A\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \cdot A^T\mathbf{y}$ to show that $N(A)^\perp = R(A^T)$.
- Solve Least Squares Problems, including "best fit" curves.
- Given an orthonormal basis $\{\mathbf{e}_i\}$ and vectors $\mathbf{x} = \sum a_i\mathbf{e}_i$, $\mathbf{y} = \sum b_i\mathbf{e}_i$, find $\langle \mathbf{x}, \mathbf{y} \rangle$, $\mathrm{dist}(\mathbf{x}, \mathbf{y})$, $\cos\theta$, $\mathrm{Proj}_{\mathbf{y}}(\mathbf{x})$, and $\mathrm{Proj}_S(\mathbf{x})$ for a subspace S.
- Calculate norms, distances, angles and projections for a given inner product (e.g. an L^2 inner product on $\mathcal{C}(a, b)$ or the Frobenius inner product on $\mathbb{R}^{m\times n}$).
- Apply the Gram-Schmidt process to a given set of vectors $\{\mathbf{x}_1, \dots \mathbf{x}_n\}$.

4. Proofs. There will be at least one short proof on the exam.

5. Review Problems.

Chapter Test B, page 200, Problems 9 and 10.

Chapter Test A, page 285, Problems 1-3, 7.

Chapter Test B, page 285-6, Problems 1-4, 6-9, 11, 12.