Math 309, Section 2 – Exam 2 Review

Exam 2 will be given in class, **Wednesday, Oct. 24**. The exam covers the material in Sections 2.1–2.3, 3.3–3.6, and 4.1–4.2 of the textbook, plus the handout for Section 4.2. You should also know the material and notation presented in class.

What to study:

1. Definitions. You should be able to give *precise* definitions of the following terms:

Determinant	minor and cofactor	Cramer's Rule for $A\mathbf{x} = \mathbf{b}$
linearly independent	linearly dependent	$\text{span}(\mathbf{v}_1, \dots \mathbf{v}_n)$
basis	dim V	Null space $N(A)$
Row space	Column space	rank(A) and nullity(A)
Standard bases of $\mathbb{R}^n$, $\mathbf{P}_n$, $\mathbb{R}^{m\times n}$.	The coordinate vector $[\mathbf{x}]_B$ of a vector in a basis B	Transition matrix U_{BA} from A-coordinates to B-coordinates.
Transformation	Linear Transformation	Linear Operator
$\ker L$	$\text{im}\, L$ and $L(S)$	Matrix $[L]_{CD}$ of L from C basis to D basis
Dilation matrix	Rotation matrix	Reflection matrix

2. Theorems. You should know and be able to apply the lemmas and theorems given in class and in Sections 3.3–4.2 of the textbook and in the two handouts. Most important: The "Two-for-One" Lemma 3.4.3, the "Basis Construction Lemma" 3.4.4, the Rank-Nullity Theorem, and the Invertibility Theorem.

3. Proofs. There will be at least one short proof on the exam. You should be able to prove (i) simple facts about linear independence, spanning sets, and bases (ii) For a matrix A, $N(A)$ and $R(A)$ are vector subspaces. (iii) For a linear transformation, $\ker L$ and $\text{im}\, L$ are vector subspaces

4. Calculations. You should know how to

- Calculate determinants by the cofactor method and by the row reduction method.
- Check whether a matrix is singular using row reduction or determinants.
- Check if a given set of vectors is a basis by row reduction, determinants or Wronskians.
- Find the dimension of a given vector space or subspace.
- Given a matrix A, find a basis of $N(A)$, a basis for the column space, and find rank(A) and nullity(A).
- Given the coordinates of a vector in one basis, find its coordinates in another basis.

- Check if a given transformation is a linear transformation.
- Write down the matrix $A = [L]_{BC}$ for a given linear transformation L in given bases B and C.
- Write down the matrices of dilations, rotations, reflections and projections for $\mathbb{R}^2$ and $\mathbb{R}^3$.
- Given linear transformation, find $\ker L$ and $L(S)$ for a subspace S. Find its kernel and range, rank and nullity.

Schedule and HW

Day	Covered in Class	Do these HW problems
Friday	Finish Section 4.2 Section 2.3	4b, 5, 6, and 14. 2a, 2c, 2e.
Monday	Review; Problems on Section 4.2 handout	Review for Exam.
Wednesday	**Exam 2**	No homework.

Office hours:

Friday: 2–3 pm.
Monday: 2–3 pm
Tuesday: 11–12 and 1-2.

From the textbook (many answers in back of the book).

Pages	Chapter Test	Problems
111	Test A	1–5.
	Test B	1, 2, 5, 9.
166-7	Test A	1– 8
	Test B	3, 4, 6, 7, 9 (hint: apply A^{-1}), 11.
199-200	Test A	1-4, 6, 8, 9.
	Test B	1–6 (Hint for 2: $\mathbf{v}_3 = 3\mathbf{v}_1 + 2\mathbf{v}_2$), 8, 9.
285-6	Test B	1-3.

Additional Practice problems:

1. Do the following 3 vectors form a basis for $\mathbb{R}^3$?

$$\mathbf{v}_1 = (1, 1, -2) \qquad \mathbf{v}_2 = (3, 2, -4), \qquad \mathbf{v}_3 = (0, 1, 0).$$

2. Let S be the subspace of $\mathbb{R}^4$ spanned by the the vectors

$$\mathbf{u}_1 = (2, -1, 3, 1), \qquad \mathbf{u}_2 = (7, -6, 5, 2), \qquad \mathbf{u}_3 = (-3, 4, 1, 0).$$

Find a basis for S and determine its dimension.

3. Let W be a subspace of a vector space V. Prove that there is a basis $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ of V whose first k vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is a basis of W.

4. Prove that the linear system $A\mathbf{x} = \mathbf{b}$ has a most one solution for all $\mathbf{b}$ if and only if $N(A) = \{0\}$. *Hint:* Suppose that $\mathbf{x}$ and $\mathbf{y}$ are both solutions. Consider $A(\mathbf{x} - \mathbf{y})$.

5. Let $L : V \to W$ be a linear transformation.

1. Define $\ker L$ and prove that it is a subspace of V.
2. Define [$\operatorname{im} L$ and prove that it is a subspace of W.