Math 309, Section 2 – Exam 1 Review

Exam 1 will be given in class on Friday, September 21. The exam covers the material in the textbook in Sections 1.1-1.5, 3.1, and the part of Section 3.2 on subspaces and the null spae of a matrix (but not the part on spanning sets).

What to study:

0. Notation. You should be familiar with the notation and terms that the book uses routinely, including: the coefficient matrix and augmented matrix of a linear system, the vector spaces $\mathbb{R}^n$, $\mathbf{P}_n$, $\mathbb{R}^{m\times n}$ and $\mathcal{C}[a,b]$. You should, of course, know how to multiply matrices and the properties of matrix multiplication, including the formulas:

$$(AB)_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj}, \qquad (AB)^{-1} = B^{-1}A^{-1}, \qquad (AB)^T = B^T A^T.$$

1. Definitions. You should be able to give *precise* definitions of the following terms:

Caution: For each term, find the precise definition in the textbook. Copy it over several times to learn it. Be sure to include all hypotheses (e.g. "If A is an $n \times n$ matrix") and all logical quantifiers (e.g. "for all", "for some", "there exists", etc.).

linear system	solution set	elementary row operations
consistent system	row echelon form	reduced row echelon form
Identity matrix I_n	inverse of a matrix	transpose of a matrix
elementary matrices (3 types)	singular matrix	non-singular matrix
Vector space (full definition)	subspace	Null space of a matrix

2. Theorems. You should know and be able to apply the lemmas and theorems given in class and in Chapters 1 and on pages 112-122 of the textbook.

3. Calculations. You should know how to

- Find the solution set of a linear system.
- Multiply matrices, simplify and solve matrix equations.
- Use row reduction to check whether a given matrix is singular or non-singular.
- Find the inverse of a matrix (by the formula for 2×2, and by row reduction for larger matrices).

4. Proofs. There will be several proofs on the exam. One will be a short proof based on the axioms of a vector space, like Lemmas 1-12 on the handout. Others may involve quoting known facts (Lemmas or Theorems) from class or from the textbook.

Review Problems for Exam 1

Go through your homework assignments, asking yourself if you could do a similar problem on an exam. Identify and focus on those types of problems that gave you the most difficulty. Find similar problems in the book and try those.

Bear in mind that, because of time constraints, problems on exams cannot be too long or too complicated.

Additional Problems:

From pages 84-86 of the textbook:

Chapter Test A: Read the bottom of page 84, do Problems 1-15 (answers in the back of the book).
Chapter Test B: Problems 1, 4, 6, 9 (compute $(AB)^T$).

From pages 166-7 of the textbook:

Chapter Test A: Problems 1-5.
Chapter Test B: Problems 1, 2, 5.