Local Max/Min Values

- A **critical point** of $f(x, y)$ is a point (x, y) where either $\nabla f(x, y) = \langle 0, 0\rangle$ or where one of the partial derivatives doesn't exist (meaning $\nabla f(x, y)$ is undefined).
- Local maximum and minimum values occur at critical points.
- The **Hessian Determinant** is

$$D(x, y) = f_{xx}f_{yy} - f_{xy}^2$$

- The **Second Derivative Test** says that if $\nabla f(x, y) = \langle 0, 0\rangle$, then you can use the value of $D(x, y)$ to tell if it's a local maximum, local minimum, or neither:
  - If $D(x, y) = 0$, then the test does not apply.
  - If $D(x, y) < 0$, then (x, y) is neither a maximum nor a minimum.
  - If $D(x, y) > 0$, then check whether $f_{xx}(x, y)$ is positive or negative. Positive means local minimum, negative means local maximum.

Absolute (Global) Max/Min Values

- The **Extreme Value Theorem** says that if R is a closed and bounded region, and if $f(x, y)$ is continuous on R, then $f(x, y)$ has a global/absolute maximum and a global/absolute minimum on R.
- The global maximum and minimum values occur either at critical points in R or on the boundary of R.

Double Integrals

- If R is a region in the x, y plane, then

$$\iint_R f(x, y)dA$$

  represents the volume underneath the graph $z = f(x, y)$ which lies above the region R.

Computing Double Integrals

- **Fubini's Theorem** says that the double integral $\iint_R f(x, y)dA$ over the rectangle $a \le x \le b$, $c \le y \le d$ can be computed using either of the "iterated integrals":

$$\int_c^d \int_a^b f(x, y)dx\,dy \quad \text{or} \quad \int_a^b \int_c^d f(x, y)dy\,dx$$

- More generally, if R is not a rectangle, but instead is the region between two curves $y = g(x)$ and $y = h(x)$, then the iterated integral is:

$$\int_a^b \int_{g(x)}^{h(x)} f(x, y)dy\,dx$$