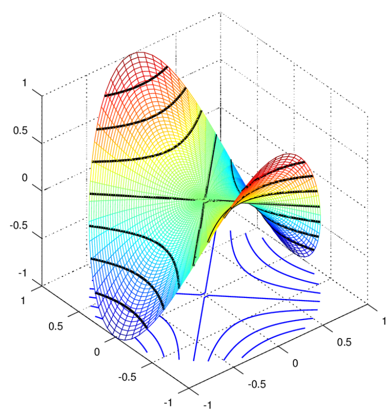
Level Curves

For a function $f(x, y)$:

- A "**level set**" (or "**level curve**") is the solution set of an equation $f(x, y) = c$, where c is some constant.
- A "**contour plot**" is a picture (in the domain) consisting of multiple level curves of f.



Limits

- The statement

$$\lim_{(x,y)\to(a,b)} f(x, y) = L$$

means that $f(x, y)$ is close to L as long as (x, y) is close enough to (a, b).

- The limit of $f(x, y)$ as (x, y) approaches (a, b) only exists if the limit is independent of the path approaching the point (a, b).
- The function $f(x, y)$ is "**continuous**" at (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$$

Partial Derivatives

Let $f(x, y)$ be a function of two variables, and (a, b) a point in the domain of f.

- $\dfrac{\partial f}{\partial x}(a, b) = f_x(a, b) = \lim\limits_{h\to 0} \dfrac{f(a + h, b) - f(a, b)}{h}$
- $\dfrac{\partial f}{\partial y}(a, b) = f_y(a, b) = \lim\limits_{h\to 0} \dfrac{f(a, b + h) - f(a, b)}{h}$

In practice, to compute the partial derivative with respect to some variable, treat all other variables as constants, and take the usual derivative.

Second Partial Derivatives

- $f_{xx} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial f}{\partial x}\right)$
- $f_{xy} = \dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial x}\right)$
- Similarly for f_{yy} and f_{yx}...
- Second partial derivatives do not depend on the order. That is, $f_{xy} = f_{yx}$.