Derivatives of Vector Functions

For a curve $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$:

- The derivative is $\vec{r}\,'(t) = \langle x'(t), y'(t), z'(t)\rangle$.
- $\vec{r}\,'(c)$ is tangent to the curve at the point $\vec{r}(c)$.
- The **unit tangent vector** is the normalization:

$$\vec{T}(c) = \frac{\vec{r}(c)}{|\vec{r}(c)|}$$

- The tangent line to the curve at the point $\vec{r}(c)$ is

$$\vec{L}(t) = \vec{r}(c) + t\,\vec{r}\,'(c)$$

Integrals of Vector Functions

For a curve $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$:

- The indefinite integral (antiderivative) is

$$\int \vec{r}(t)dt = \left\langle \int x(t)dt, \int y(t)dt, \int z(t)dt \right\rangle$$

- The definite integral from $t = a$ to $t = b$ is

$$\int_a^b \vec{r}(t)dt = \left\langle \int_a^b x(t)dt, \int_a^b y(t)dt, \int_a^b z(t)dt \right\rangle$$

Physics

If we denote position $\vec{r}(t)$, velocity $\vec{v}(t)$, and acceleration $\vec{a}(t)$:

- $\vec{v}(t) = \vec{r}\,'(t)$
- $\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$

Alternatively:

- $\vec{r}(t) = \displaystyle\int \vec{v}(t)dt$
- $\vec{v}(t) = \displaystyle\int \vec{a}(t)dt$

Arc Length

- The length of a parametric curve $\vec{r}(t)$ from $t = a$ to $t = b$ is given by

$$\int_a^b |\vec{r}\,'(t)|dt$$

or

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$

- The arc-length function (starting at $t = c$) is given by

$$s(t) = \int_c^t |\vec{r}\,'(u)|du$$