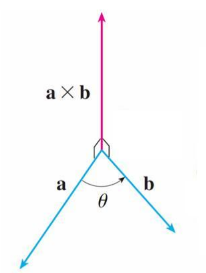
The Cross Product

- For vectors $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$, their cross product is

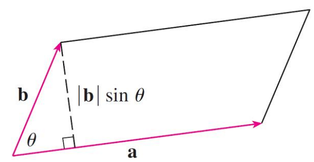
$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$$
$$= \langle a_2 b_3 - b_2 a_3,\ a_3 b_1 - b_3 a_1,\ a_1 b_2 - b_1 a_2 \rangle$$

- $\vec{a} \times \vec{b}$ is always orthogonal (perpendicular) to both $\vec{a}$ and $\vec{b}$.
- $\vec{i} \times \vec{j} = \vec{k}$, $\vec{j} \times \vec{k} = \vec{i}$, and $\vec{k} \times \vec{i} = \vec{j}$
- Geometrically, the cross product $\vec{a} \times \vec{b}$ follows the "right-hand rule". To visualize, if you are looking from the tip of the arrow $\vec{a} \times \vec{b}$, then you should have to rotate $\vec{a}$ counter-clockwise to get to $\vec{b}$.

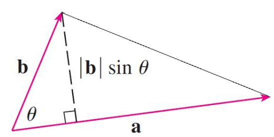


Area Formulas

- The norm of the cross product $\vec{a} \times \vec{b}$ is the number $|\vec{a}||\vec{b}|\sin(\theta)$, where θ is the angle between $\vec{a}$ and $\vec{b}$ (as in the picture)
- $|\vec{a}||\vec{b}|\sin(\theta)$ is also the area of a parallelogram with side lengths $|\vec{a}|$ and $|\vec{b}|$ which make an angle θ with each other.

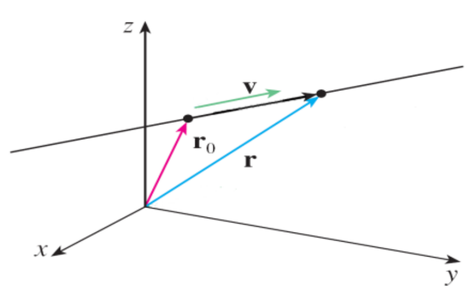


- $\frac{1}{2}|\vec{a}||\vec{b}|\sin(\theta)$ is also the area of a triangle with sides of length $|\vec{a}|$ and $|\vec{b}|$ which meet at an angle of θ. (This is just half of the above parallelogram)



Lines The line in $\mathbb{R}^3$ which goes through the point $\vec{r}_0 = \langle a, b, c \rangle$ and in the direction $\vec{v} = \langle v_1, v_2, v_3 \rangle$ is given by the parameterization

$$\vec{r}(t) = \vec{r}_0 + t\vec{v} = \langle a + v_1 t,\ b + v_2 t,\ c + v_3 t \rangle$$



Planes
The points $\langle x, y, z \rangle$ on the plane containing the point $\vec{P} = \langle a, b, c \rangle$ with normal vector $\vec{n} = \langle n_1, n_2, n_3 \rangle$ satisfy the equation:

$$n_1(x - a) + n_2(y - b) + n_3(z - c) = 0$$