Divergence

- The divergence of a vector field $\vec{F} = \langle P, Q, R\rangle$ is

$$\text{div}(\vec{F}) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

- This is often written as $\text{div}(\vec{F}) = \nabla \cdot \vec{F}$.

Curl

- The curl of a vector field $\vec{F} = \langle P, Q, R\rangle$ is

$$\text{curl}(\vec{F}) = \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle$$

- This is often written as $\text{curl}(\vec{F}) = \nabla \times \vec{F}$.
- If $\vec{F}$ is conservative, then $\nabla \times \vec{F} = \vec{0}$. In other words, for a function f, we have

$$\nabla \times (\nabla f) = \vec{0}$$

Surface Area of Graphs

- The surface area of a graph $z = f(x, y)$ which lies over the region D in the x, y plane is

$$\text{Area} = \iint_D \sqrt{1 + f_x^2 + f_y^2}\, dA$$

Surface Area for Parametric Surfaces

For the following, consider a parameterized surface

$$\vec{r}(u, v) = \langle x(u, v),\ y(u, v),\ z(u, v)\rangle$$

- Define the vectors $\vec{r}_u$ and $\vec{r}_v$ to be

$$\vec{r}_u = \left\langle \frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right\rangle$$

$$\vec{r}_v = \left\langle \frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v} \right\rangle$$

- The surface area of the region described by (u, v) in some region D of the u, v plane is

$$\text{Area} = \iint_D |\vec{r}_u \times \vec{r}_v|\, dA$$