Area, Density, and Mass

- For a region D in the x, y plane, $\iint_D dA$ gives the area of D.
- If $\rho(x, y)$ is the density (mass per area) on a region D, then $\iint_D \rho(x, y)\, dA$ gives the mass of D.

Double Integrals in Polar Coordinates

- The integral $\iint_D f(x, y)\, dA$, written in polar coordinates, is given by

$$\iint_{\widetilde{D}} f(r\cos(\theta), r\sin(\theta))\, r\, dr\, d\theta$$

where $\widetilde{D}$ is the region D expressed in polar coordinates.

Triple Integrals

- **Fubini's Theorem** says that if $f(x, y, z)$ is continuous on the rectangle E, given by $a \leq x \leq b$, $c \leq y \leq d$, $g \leq z \leq h$, then the triple integral $\iiint_E f(x, y, z)\, dV$ can be computed by an "iterated integral" of the form

$$\int_g^h \int_c^d \int_a^b f(x, y, z)\, dx\, dy\, dz$$

- There are six possible re-orderings of the three iterated integrals, and they are all equal.
- If E is the space between two surfaces $z = g(x, y)$ and $z = h(x, y)$, and if D is the shadow/projection of E onto the x, y plane, then

$$\iiint_E f(x, y, z) dV = \iint_D \int_{g(x,y)}^{h(x,y)} f(x, y, z) dz dA$$

Volume, Density, and Mass

- For a region E in 3-dimensional space, $\iiint_E dV$ gives the volume of E.
- If $\rho(x, y, z)$ is the density (mass per volume) on a region E, then $\iiint_E \rho(x, y, z)\, dV$ gives the mass of E.