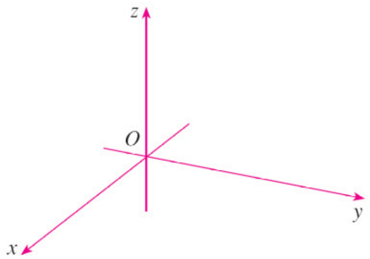
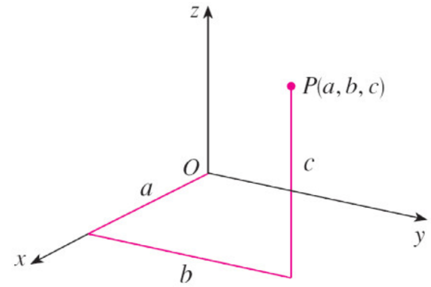
Right-Hand Coordinate System



Plotting Points in 3D



Distance Formulas

- For $P = (p_1, p_2, p_3)$ and $Q = (q_1, q_2, q_3)$,

$$|PQ| = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2 + (p_3 - q_3)^2}$$

- The sphere centered at $P = (a, b, c)$ with radius r consists of the points (x, y, z) so that:

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$

Vector Formulas

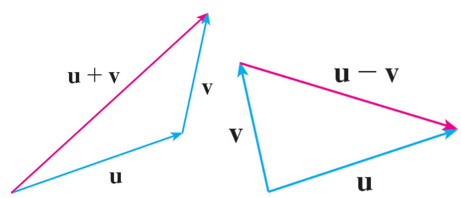
- The length of $\overrightarrow{PQ}$ (written $|\overrightarrow{PQ}|$) is just $|PQ|$. (same as for line segment)
- The unit vector pointing in the direction $\vec{v}$ is $\dfrac{\vec{v}}{|\vec{v}|}$. (scale the vector by the reciprocal of its norm)

Algebraic Vector Arithmetic

Suppose that $\vec{v} = \langle v_1, v_2, v_3\rangle$ and $\vec{w} = \langle w_1, w_2, w_3\rangle$, and c is a scalar.

- $\vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2, v_3 + w_3\rangle$
- $\vec{v} - \vec{w} = \langle v_1 - w_1, v_2 - w_2, v_3 - w_3\rangle$
- $c\,\vec{v} = \langle cv_1, cv_2, cv_3\rangle$

Geometric Vector Arithmetic



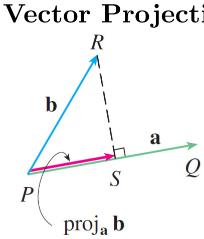
The Dot Product

- $\vec{v} \cdot \vec{w} = v_1w_1 + v_2w_2 + v_3w_2$
- $\vec{v} \cdot \vec{v} = |\vec{v}|^2$
- $\vec{v} \cdot \vec{w} = |\vec{v}|\,|\vec{w}|\,\cos(\theta)$

  or $\quad \cos(\theta) = \dfrac{\vec{v} \cdot \vec{w}}{|\vec{v}|\,|\vec{w}|}$

  (θ is the angle between $\vec{v}$ and $\vec{w}$)
- $\vec{v} \cdot \vec{w} = 0$ if and only if $\vec{v}$ and $\vec{w}$ are orthogonal.

Vector Projections



- $\text{comp}_{\vec{a}}(\vec{b}) = |\text{proj}_{\vec{a}}(\vec{b})| = |\vec{b}|\cos(\theta) = \vec{b} \cdot \dfrac{\vec{a}}{|\vec{a}|}$
- $\text{proj}_{\vec{a}}(\vec{b}) = \text{comp}_{\vec{a}}(\vec{b})\,\dfrac{\vec{a}}{|\vec{a}|} = \left(\vec{b} \cdot \dfrac{\vec{a}}{|\vec{a}|}\right)\dfrac{\vec{a}}{|\vec{a}|}$