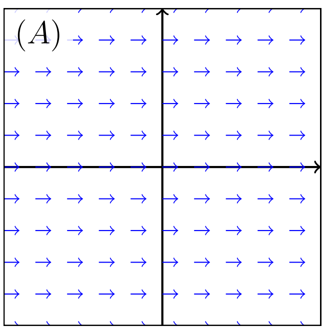
Name: ____

Clear your desk of everything excepts pens, pencils and erasers.
If you have a question, please raise your hand.

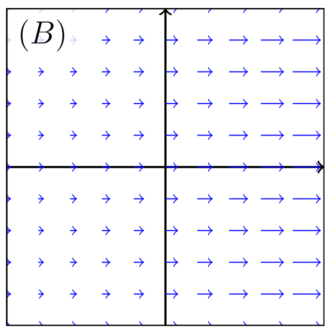
1. (2 points) Match each vector field $\vec{F}(x,y)$ with its picture below.

   $\underline{(D)}$ $\vec{F}(x,y) = \langle x, -y\rangle$

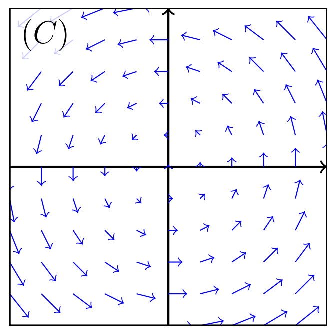
   $\underline{(B)}$ $\vec{F}(x,y) = \langle e^x, 0\rangle$



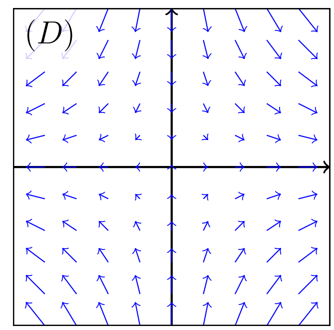
$\langle 1, 0\rangle$



$\langle e^x, 0\rangle$



$\langle -y, x\rangle$

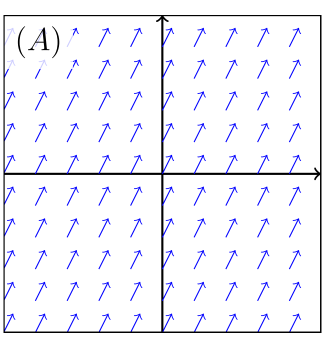


$\langle x, -y\rangle$

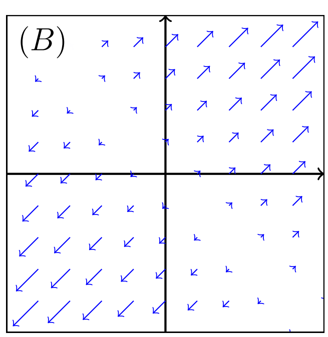
2. (2 points) Match each function $f(x,y)$ with the picture of its gradient vector field, $\nabla f(x,y)$.

   $\underline{(A)}$ $f(x,y) = x + 2y$

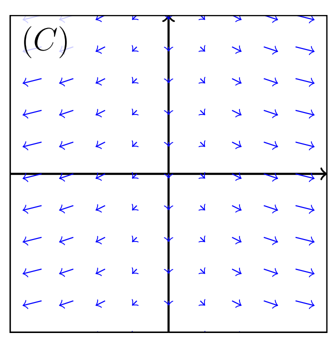
   $\underline{(C)}$ $f(x,y) = x^2 - y$



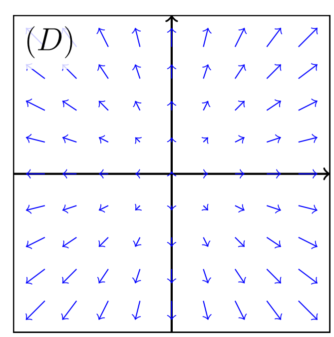
$x + 2y$



$(x+y)^2$



$x^2 - y$



$x^2 + y^2$

Continue on to back side

3. (2 points) Compute the line integral $\int_C f(x,y)\,ds$, where $f(x,y) = xy$, and C is the arc of the unit circle from $(1,0)$ to $(0,1)$, traversed counter-clockwise.

Solution: The curve C can be parameterized by $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$ for $0 \leq t \leq \frac{\pi}{2}$. Then $\vec{r}\,'(t) = \langle -\sin(t), \cos(t) \rangle$. So the line integral becomes:

$$\begin{aligned}\int_C xy\,ds &= \int_0^{\pi/2} \cos(t)\sin(t)\sqrt{\sin^2(t)+\cos^2(t)}\,dt \\ &= \int_0^{\pi/2} \cos(t)\sin(t)\,dt \\ &= \frac{1}{2}\left[\sin^2(t)\right]_0^{\pi/2} \\ &= \frac{1}{2}\end{aligned}$$