**Name**: ______________________________________________

Show your work, or give reasoning, to receive full credit.

1. (3 points) The integral

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} F(x,y,z)dz\,dy\,dx$$

can be re-written as

$$\int_{a}^{b}\int_{u(z)}^{v(z)}\int_{f(y,z)}^{g(y,z)} F(x,y,z)dx\,dy\,dz$$

What are the bounds on the new integral?

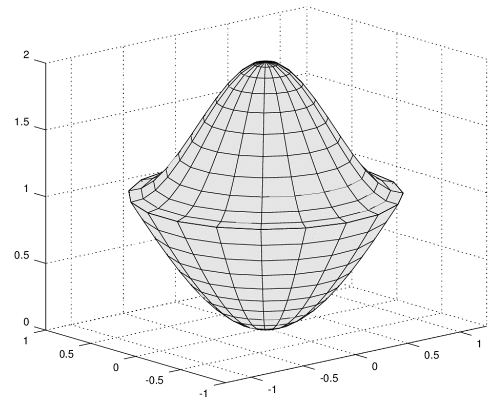
$a = \underline{0}$, $b = \underline{1}$

$u(z) = \underline{0}$, $v(z) = \underline{1 - z}$

$f(y,z) = \underline{-\sqrt{y}}$, $g(y,z) = \underline{\sqrt{y}}$

Continue on to back side

**Bonus**. (1 pt) Consider the following surface.



The top (above $z = 1$) is the graph of $z = 1 + \cos^2\left(\frac{\pi}{2}\sqrt{x^2+y^2}\right)$ and the bottom (below $z = 1$) is the graph of $z = 1 - \cos\left(\frac{\pi}{2}\sqrt{x^2+y^2}\right)$. Find the volume inside this surface.

**Solution:** The surfaces intersect in the circle of radius 1 in the plane $z = 1$. So the projection of the region onto the $x, y$-plane is the interior of the unit disc. Then the volume is given by the integral over that region of the top minus the bottom. The top minus the bottom is given by

$$1 + \cos^2\left(\frac{\pi}{2}\sqrt{x^2+y^2}\right) - \left(1 - \cos\left(\frac{\pi}{2}\sqrt{x^2+y^2}\right)\right) = \cos^2\left(\frac{\pi}{2}\sqrt{x^2+y^2}\right) + \cos\left(\frac{\pi}{2}\sqrt{x^2+y^2}\right)$$

So the volume is

$$\begin{aligned}\iint_D \left(\cos^2\left(\frac{\pi}{2}\sqrt{x^2+y^2}\right) + \cos\left(\frac{\pi}{2}\sqrt{x^2+y^2}\right)\right) dA &= \int_0^1 \int_0^{2\pi} \left(\cos^2\left(\frac{\pi}{2}r\right) + \cos\left(\frac{\pi}{2}r\right)\right) r\,d\theta\,dr \\ &= 2\pi \int_0^1 \left(\cos^2\left(\frac{\pi}{2}r\right) + \cos\left(\frac{\pi}{2}r\right)\right) r\,dr\end{aligned}$$

Let's do the two integrals separately. For the second one:

$$\begin{aligned}\int_0^1 r\cos\left(\frac{\pi}{2}r\right)\,dr &= \frac{2}{\pi}\left(\left[r\sin\left(\frac{\pi}{2}r\right)\right]_0^1 - \int_0^1 \sin\left(\frac{\pi}{2}r\right)dr\right) \qquad \text{(int. by parts)} \\ &= \frac{2}{\pi}\left(1 - \int_0^1 \sin\left(\frac{\pi}{2}r\right)dr\right) \\ &= \frac{2}{\pi}\left(1 + \frac{2}{\pi}\left[\cos\left(\frac{\pi}{2}r\right)\right]_0^1\right) \\ &= \frac{2}{\pi}\left(1 - \frac{2}{\pi}\right)\end{aligned}$$

Now for the first term:

$$\begin{aligned}
\int_0^1 r\cos^2\left(\frac{\pi}{2}r\right)\,dr &= \frac{1}{2}\int_0^1 r\left(1+\cos(\pi r)\right)\,dr \\
&= \frac{1}{2}\left(\int_0^1 r\,dr + \int_0^1 r\cos(\pi r)\,dr\right) \\
&= \frac{1}{2}\left(\frac{1}{2} + \int_0^1 r\cos(\pi r)\,dr\right) \\
&= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{\pi}\left[r\sin(\pi r)\right]_0^1 - \frac{1}{\pi}\int_0^1 \sin(\pi r)dr\right) && \text{(int. by pts.)} \\
&= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{\pi}\int_0^1 \sin(\pi r)dr\right) \\
&= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{\pi^2}\left[\cos(\pi r)\right]_0^1\right) \\
&= \frac{1}{2}\left(\frac{1}{2} - \frac{2}{\pi^2}\right) \\
&= \frac{1}{4} - \frac{1}{\pi^2}
\end{aligned}$$

Putting it all together, the volume is

$$\begin{aligned}
\text{volume} &= 2\pi\int_0^1 \left(\cos^2\left(\frac{\pi}{2}r\right) + \cos\left(\frac{\pi}{2}r\right)\right) r\,dr \\
&= 2\pi\left(\frac{1}{4} - \frac{1}{\pi^2} + \frac{2}{\pi}\left(1 - \frac{2}{\pi}\right)\right) \\
&= \frac{\pi}{2} + \frac{2}{\pi} + 4\left(1 - \frac{2}{\pi}\right) \\
&= \frac{\pi}{2} + \frac{2}{\pi} + 4 - \frac{8}{\pi} \\
&= \frac{\pi}{2} - \frac{6}{\pi} + 4
\end{aligned}$$