Name: ______________________________

Show your work, or give reasoning, to receive full credit.

1. (2 points) Compute the volume of the tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(0,3,0)$, and $(0,0,5)$, using a double integral.

Solution: The "top" of the tetrahedron is the triangle through the 3 points $(2,0,0)$, $(0,3,0)$, and $(0,0,5)$, which is in the plane

$$z = \frac{5}{6}(6-3x-2y)$$

We only want the volume under this plane where x, y, z are all positive. So the x and y coordinates are within the triangle with vertices $(0,0)$, $(2,0)$, and $(0,3)$. This is the domain of integration, D. The top edge of this triangle is the line $y = \frac{3}{2}(2-x)$. So the volume is given by

$$\begin{aligned}
\iint_D \frac{5}{6}(6-3x-2y)\,dA &= \frac{5}{6}\int_0^2 \int_0^{\frac{3}{2}(2-x)} (6-3x-2y)\,dy\,dx \\
&= \frac{5}{6}\int_0^2 \left[(6-3x)y - y^2\right]_0^{\frac{3}{2}(2-x)}\,dx \\
&= \frac{5}{6}\int_0^2 \left[3(2-x)y - y^2\right]_0^{\frac{3}{2}(2-x)}\,dx \\
&= \frac{5}{6}\int_0^2 \frac{9}{4}(2-x)^2 dx \\
&= \frac{15}{8}\int_0^2 (2-x)^2 dx \\
&= 5
\end{aligned}$$

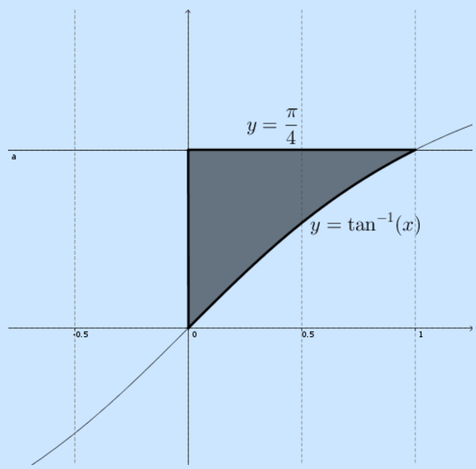
Continue on to back side

2. The order of integration for the integral $\int_0^1 \int_{\tan^{-1}(x)}^{\pi/4} F(x,y)\,dy\,dx$ can be reversed to give the integral

$$\int_a^b \int_{f(y)}^{g(y)} F(x,y)\,dx\,dy$$

What are the bounds of the new integrals?

Solution: The region D of integration looks like



Notice that the region D lies between $y = 0$ and $y = \pi/4$. For a given y, the x values are between 0 and $x = \tan(y)$. So the integral can be switched to

$$\int_0^{\pi/4} \int_0^{\tan(y)} F(x,y)\,dx\,dy$$