Name: ____________________

Show your work, or give reasoning, to receive full credit.

1. Consider the function $f(x, y) = \tan^{-1}(e^{xy})$.

   (a) (1 point) Compute the gradient $\nabla f(x, y)$.

   Solution:

$$\frac{\partial f}{\partial x} = \frac{ye^{xy}}{1+e^{2xy}}$$
$$\frac{\partial f}{\partial y} = \frac{xe^{xy}}{1+e^{2xy}}$$

   So the gradient vector is

$$\nabla f(x, y) = \left\langle \frac{ye^{xy}}{1+e^{2xy}}, \frac{xe^{xy}}{1+e^{2xy}} \right\rangle$$

   (b) (1 point) Compute the directional derivative $D_{\vec{u}}f$ at the point $(1, \ln(2))$, where $\vec{u}$ is the unit vector in the direction $\langle -1, \ln(2)\rangle$.

   Solution: Call the given vector $\vec{v} = \langle -1, \ln(2)\rangle$. It is not a unit vector, so we should divide by its length to get a unit vector $\vec{u} = \frac{1}{|\vec{v}|}\vec{v}$. We also need the value of the gradient at the point $(1, \ln(2))$:

$$\nabla f(1, \ln(2)) = \frac{2}{5}\langle \ln(2), 1\rangle$$

   Finally, the directional derivative is the dot product:

$$\begin{aligned} D_{\vec{u}}f(1, \ln(2)) &= \vec{u} \cdot \nabla f(1, \ln(2)) \\ &= \frac{2}{5|\vec{v}|}\langle -1, \ln(2)\rangle \cdot \langle \ln(2), 1\rangle \\ &= \frac{2}{5|\vec{v}|}(-\ln(2) + \ln(2)) \\ &= 0 \end{aligned}$$

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