Name: ____________________

Clear your desk of everything excepts pens, pencils and erasers.
If you have a question, please raise your hand.

1. Consider the function $f(x, y) = x^2 + 2y^2$.

   (a) (1 point) Compute $f_x(x, y)$.

   Solution: $f_x(x, y) = 2x$

   (b) (1 point) Evaluate $f_x(1, 1)$.

   Solution: $f_x(1, 1) = 2(1) = 2$

   (c) (1 point) Compute $f_y(x, y)$.

   Solution: $f_y(x, y) = 4y$

   (d) (1 point) Evaluate $f_y(1, 1)$.

   Solution: $f_y(1, 1) = 4(1) = 4$

Continue on to back side

(e) (1 point) Write the equation of the tangent plane to the graph of $z = f(x, y)$ at the point where $x = 1$ and $y = 1$.

Solution: The tangent plane at $(x, y) = (1, 1)$ is

$$\begin{aligned} z &= f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1) \\ &= 3 + 2(x-1) + 4(y-1) \end{aligned}$$

(f) (1 point) Suppose that $x = t^2 + 5$ and $y = e^t \sin(t)$. Then what is $\frac{df}{dt}$?

Solution: According to the chain rule:

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

The partial derivatives of f are:

$$\frac{\partial f}{\partial x} = 2x = 2t^2 + 10$$

$$\frac{\partial f}{\partial y} = 4y = 4e^t \sin(t)$$

The derivatives of x and y are:

$$\frac{dx}{dt} = 2t$$

$$\frac{dy}{dt} = e^t(\sin(t) + \cos(t))$$

So all together, we have:

$$\frac{df}{dt} = (2t^2 + 10) \cdot (2t) + (4e^t \sin(t)) \cdot e^t(\sin(t) + \cos(t))$$