Name: ____

Show your work, or give reasoning, to receive full credit.

1. Consider the function $f(x,y) = e^{2x}\sin(x+y^2)$.

   (a) (1 point) Compute the partial derivative $f_x = \frac{\partial f}{\partial x}$.

   Solution: $\dfrac{\partial f}{\partial x} = 2e^{2x}\sin(x+y^2) + e^{2x}\cos(x+y^2)$

   This can also be factored and written as

   $$e^{2x}\left(2\sin(x+y^2) + \cos(x+y^2)\right)$$

   (b) (1 point) Compute the second partial derivative $f_{xy} = \frac{\partial^2 f}{\partial y \partial x}$.

   Solution: $\dfrac{\partial^2 f}{\partial y \partial x} = 4ye^{2x}\cos(x+y^2) - 2ye^{2x}\sin(x+y^2)$

   This can also be factored and written as

   $$2ye^{2x}\left(2\cos(x+y^2) - \sin(x+y^2)\right)$$

Continue on to back side

2. (2 points) Compute the following limit, or show that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$$

Solution: The limit does not exist, so we will show that there are at least two different paths approaching $(0,0)$ which give different limits. Let's approach along an arbitrary line $y = mx$:

$$\begin{aligned}\lim_{x\to 0} \frac{(mx)^2}{x^2+(mx)^2} &= \lim_{x\to 0} \frac{m^2x^2}{(1+m^2)x^2} \\ &= \lim_{x\to 0} \frac{m^2}{1+m^2} \\ &= \frac{m^2}{1+m^2}\end{aligned}$$

The limit depends on m, so the limit does not exist.

For a more straightforward solution, you could just compute two different limits. For example, the limit along $y = x$ would be $\frac{1}{2}$, but the limit along $y = 2x$ would be $\frac{4}{5}$.