Name: ____

Clear your desk of everything excepts pens, pencils and erasers.
If you have a question, please raise your hand.

1. (2 points) Which of the following describes the domain of the function $f(x, y) = \dfrac{e^{x+y}}{x-y}$?

Solution: The exponential gives no restrictions. The only restriction is that the denominator $(x-y)$ cannot be zero. So the domain includes all points except those for for which $x = y$. In other words, the domain is all (x, y) such that $x \neq y$.

2. (2 points) What is the limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{y^2 - x^2}{y - x}$?

Solution: Notice that the numerator factors as $y^2 - x^2 = (y - x)(y + x)$. So this limit is equal to

$$\lim_{(x,y)\to(0,0)} (y + x) = 0 + 0 = 0$$

Continue on to back side

3. (3 points) Compute the length of the curve

$$\vec{r}(t) = \left\langle t,\ \frac{2\sqrt{2}}{3}t^{3/2},\ \frac{1}{2}t^2 \right\rangle$$

from $t = 0$ to $t = 1$.

Solution: We need to integrate the "speed" of this curve (meaning $|\vec{r}\,'(t)|$). So let's compute the derivative:

$$\vec{r}\,'(t) = \left\langle 1,\ \sqrt{2t},\ t \right\rangle$$

The norm of this vector is:

$$|\vec{r}\,'(t)| = \sqrt{1 + 2t + t^2} = \sqrt{(1+t)^2} = 1 + t$$

The length of the curve from $t = 0$ to $t = 1$ is then:

$$\int_0^1 |\vec{r}\,'(t)|\, dt = \int_0^1 (1+t)\, dt = \left[t + \frac{t^2}{2} \right]_0^1 = \frac{3}{2}$$

4. (3 points) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.

Solution: We must show that there are at least two different paths, both of which approach $(0,0)$, but give different limits. Let's approach the origin along a generic line $y = mx$ for some constant m. So we plug in $y = mx$ in the original expression and take the 1-variable limit as $x \to 0$.

$$\lim_{x\to 0} \frac{x(mx)}{x^2 + (mx)^2} = \lim_{x\to 0} \frac{mx^2}{(1+m^2)x^2} = \frac{m}{1+m^2}$$

This depends on the slope m. In particular, for $m = 1$ ($y = x$) the limit is $\frac{1}{2}$, but for $m = 2$ ($y = 2x$) the limit is $\frac{2}{5}$. So the limit does not exist.

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