Name: ____________________

Show your work, or give reasoning, to receive full credit.

1. Consider the parametric curve
$$\vec{r}(t) = \left\langle \ln(t),\ t^2 \sin(\pi t),\ 5t - 3 \right\rangle$$

   (a) (1 point) Compute $\vec{r}\,'(t)$.

   Solution: $\vec{r}\,'(t) = \left\langle \dfrac{1}{t},\ 2t\sin(\pi t) + \pi t^2 \cos(\pi t),\ 5 \right\rangle$

   (b) (1 point) Evaluate $\vec{r}\,'(1)$.

   Solution: $\vec{r}\,'(1) = \langle 1, -\pi, 5\rangle$

   (c) (2 points) Write an equation for the tangent line to the curve at the point where $t = 1$.

   Solution: First note that $\vec{r}(1) = \langle 0, 0, 2\rangle$. The direction is the vector from the previous part. So the tangent line equation is
$$\vec{\ell}(t) = \langle 0, 0, 2\rangle + t\,\langle 1, -\pi, 5\rangle$$

2. (1 point) For the curve
$$\vec{r}(t) = \langle 2\cos(t),\ 2\sin(t),\ 3t\rangle\,,$$
what is $\vec{T}(\pi)$, the unit tangent vector at $t = \pi$?

   Solution: The tangent vector is the derivative
$$\vec{r}\,'(t) = \langle -2\sin(t),\ 2\cos(t),\ 3\rangle$$
At $t = \pi$, this is
$$\vec{r}\,'(\pi) = \langle 0, -2, 3\rangle$$
The length of this vector is $\sqrt{4+9} = \sqrt{13}$, so the unit tangent vector is
$$\vec{T}(\pi) = \frac{1}{\sqrt{13}}\,\langle 0, -2, 3\rangle$$

Continue on to back side

3. (1 point) Assume that at time $t = 0$, a moving object is at position $\langle 0, 0, 1\rangle$, and that its velocity is given by

$$\vec{v}(t) = \left\langle \frac{1}{1+t^2},\ 2t,\ e^{3t} \right\rangle$$

What is the position of the object at time $t = 1$?

Solution: The position is given by

$$\vec{r}(t) = \int \vec{v}(t)dt = \left\langle \tan^{-1}(t),\ t^2,\ \frac{1}{3}e^{3t} \right\rangle + \langle c_1, c_2, c_3\rangle$$

Since $\vec{r}(0) = \langle 0, 0, 1\rangle$, we have

$$\langle 0, 0, 1\rangle = \left\langle 0, 0, \frac{1}{3} \right\rangle + \langle c_1, c_2, c_3\rangle$$

So we get the position function is

$$\vec{r}(t) = \left\langle \tan^{-1}(t),\ t^2,\ \frac{e^{3t}+2}{3} \right\rangle$$

So at time $t = 1$, the position is

$$\vec{r}(1) = \left\langle \frac{\pi}{4},\ 1,\ \frac{e^3+2}{3} \right\rangle$$

4. (Bonus 1 pt) A projectile is fired from position $\langle 0, 0\rangle$ with initial speed 64 feet per second, at an angle θ above the horizontal. If the projectile lands at position $\langle 128, 0\rangle$, then what is θ?

(Use 32 feet per second per second for the acceleration of gravity)

Solution: From class, we know that the position function is given by:

$$\vec{r}(t) = \left\langle 64\cos(\theta)\,t,\ 64\sin(\theta)\,t - 16t^2 \right\rangle$$

How long does it take for the projectile to land? Solve for when the y-coordinate is zero:

$$0 = 64\sin(\theta)\,t - 16t^2 = 16t\,(4\sin(\theta) - t)$$

The only non-zero solution is $t = 4\sin(\theta)$. At this time, the x-coordinate should be 128. Use this to finally solve for θ:

$$\begin{aligned}
128 &= 64\cos(\theta)\cdot 4\sin(\theta) \\
\frac{1}{2} &= \sin(\theta)\cos(\theta) \\
1 &= \sin(2\theta) && (\sin(2\theta) = 2\sin(\theta)\cos(\theta)) \\
\frac{\pi}{2} &= 2\theta \\
\frac{\pi}{4} &= \theta
\end{aligned}$$