Name: ____

Show your work, or give reasoning, to receive full credit.

1. (1 point) For the vector function given by

$$\vec{r}(t) = \left\langle t^2 + 1,\ \frac{t^2 - 1}{t - 1},\ e^{t-1} \right\rangle,$$

compute the limit $\lim\limits_{t \to 1} \vec{r}(t)$.

Solution: We know that this is given by

$$\lim_{t \to 1} \vec{r}(t) = \left\langle \lim_{t \to 1} t^2 + 1,\ \lim_{t \to 1} \frac{t^2 - 1}{t - 1},\ \lim_{t \to 1} e^{t-1} \right\rangle$$

The x and z components are continuous at $t = 1$, so we may simply plug in $t = 1$ to compute the limits. But the y-component is not defined for $t = 1$ (since the denominator is $t - 1$), so we cannot plug in $t = 1$. For this, note that when $t \neq 1$, we can factor and cancel:

$$\frac{t^2 - 1}{t - 1} = \frac{(t - 1)(t + 1)}{t - 1} = t + 1$$

So the y-component agrees with $t + 1$ when $t \neq 1$. This means that the limits are the same:

$$\lim_{t \to 1} \frac{t^2 - 1}{t - 1} = \lim_{t \to 1} t + 1 = 2$$

So we finally get:

$$\lim_{t \to 1} \vec{r}(t) = \langle 2, 2, 1 \rangle$$

Continue on to back side

2. (3 points) Consider the curve

$$\vec{r}(t) = \left\langle t, t^2, t^3 \right\rangle$$

At what values of t does this curve intersect the surface given by the equation $z = y^2 - x^2$?

(Hint: there are 3 points of intersection)

Solution: For a point to be in the intersection, it means its coordinates must satisfy both equations. At time t, the coordinates are given by $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle = \langle t, t^2, t^3\rangle$. On the other hand, since the point is also on the hyperbolic paraboloid, the coordinates also satisfy $z(t) = y(t)^2 - x(t)^2$. Plugging in the coordinate functions, this gives

$$\begin{aligned} t^3 &= (t^2)^2 - (t)^2 \\ t^3 &= t^4 - t^2 \\ t^3 &= t^2(t^2 - 1) \end{aligned}$$

At this point, we can see that $t = 0$ will be a solution. There are still 2 others, though. So, assuming now that $t \neq 0$, we can divide the equation by t^2:

$$\begin{aligned} t &= t^2 - 1 \\ 0 &= t^2 - t - 1 \end{aligned}$$

From here, use the quadratic formula to get that the two other solutions are

$$t = \frac{1 \pm \sqrt{5}}{2}$$

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