Name: ______________________________

Clear your desk of everything excepts pens, pencils and erasers.
If you have a question, please raise your hand.

1. (1 point) Which of the following represents the area of the triangle formed by the points P, Q, and R, where θ is the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$?

Solution: The area is given by half of the norm of the cross product of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$, which is

$$\frac{1}{2}\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PR}\right|\sin(\theta)$$

2. (1 point) Which of the following is the angle between the vectors $\langle 5, 2\rangle$ and $\langle 4, -10\rangle$?

Solution: The dot product of these vectors is

$$\langle 5, 2\rangle \cdot \langle 4, -10\rangle = 5 \cdot 4 + 2 \cdot (-10) = 20 - 20 = 0$$

Since the dot product is zero, the vectors are orthogonal, and so the angle is $\frac{\pi}{2}$.

Continue on to back side

3. (2 points) Give a unit vector which is orthogonal to both $\langle 1,0,1\rangle$ and $\langle 2,3,-1\rangle$.

Solution: The cross product will be orthogonal to both vectors:

$$\langle 1,0,1\rangle \times \langle 2,3,-1\rangle = \langle -3,3,3\rangle$$

But this is not a unit vector, so we need to rescale it. The unit vector will be

$$\frac{1}{\sqrt{27}}\langle -3,3,3\rangle$$

4. (2 points) Compute the component of $\vec{v} = \langle 1,1,1\rangle$ in the direction of $\vec{w} = \langle 1,2,0\rangle$.
(Recall that $\mathrm{comp}_{\vec{w}}(\vec{v}) = |\mathrm{proj}_{\vec{w}}(\vec{v}))$

Solution: The component is given by

$$\begin{aligned}\mathrm{comp}_{\vec{w}}(\vec{v}) &= \vec{v} \cdot \frac{\vec{w}}{|\vec{w}|}\\ &= \frac{\langle 1,1,1\rangle \cdot \langle 1,2,0\rangle}{|\langle 1,2,0\rangle|}\\ &= \frac{3}{\sqrt{5}}\end{aligned}$$