Name: ______________________________

1. (2 points) Consider the points $P = (1, 0, 0)$, $Q = (2, 3, 3)$, $R = (0, 1, 0)$, and $S = (0, 1, 1)$. Write down the unit vector (in components) pointing in the same direction as $\overrightarrow{PQ} - \overrightarrow{RS}$.

Solution: First let's compute the two vectors:

$$\overrightarrow{PQ} = \langle 2-1, 3-0, 3-0 \rangle = \langle 1, 3, 3 \rangle$$

$$\overrightarrow{RS} = \langle 0-0, 1-1, 1-0 \rangle = \langle 0, 0, 1 \rangle$$

The difference is

$$\overrightarrow{PQ} - \overrightarrow{RS} = \langle 1-0, 3-0, 3-1 \rangle = \langle 1, 3, 2 \rangle$$

The norm of this vector is

$$\left| \overrightarrow{PQ} - \overrightarrow{RS} \right| = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{14}$$

So the corresponding unit vector is

$$\frac{1}{14} \langle 1, 3, 2 \rangle = \left\langle \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}} \right\rangle$$

Continue on to back side

2. (2 points) If $|\vec{v}| = 2$, and $\vec{v} \cdot \vec{w} = \sqrt{2}$, and the angle between $\vec{v}$ and $\vec{w}$ is $\dfrac{\pi}{4}$, then what is $|\vec{w}|$?

Solution: We need to use the formula

$$\vec{v} \cdot \vec{w} = |\vec{v}|\,|\vec{w}|\,\cos(\theta)$$

Plugging in what we know, this gives

$$\sqrt{2} = 2\,|\vec{w}|\,\cos(\pi/4) = \sqrt{2}\,|\vec{w}|$$

Solving this equation, we get that $|\vec{w}| = 1$.