

2. (2 points) Multiple Choice. Circle the best answer. No partial credit available

Find the horizontal asymptote(s) of $f(x) = \frac{x^4 - 2x^2 + 1}{x^4 + x}$

- A. y = x
- **B.** y = 1
- C. y = 0
- D. There are no horizontal asymptotes
- E. None of the above

Solution: Divide the numerator and denominator by x^4 and then take the limit:

$$\lim_{x \to \infty} \frac{x^4 - 2x^2 + 1}{x^4 + x} = \lim_{x \to \infty} \frac{1 - \frac{2}{x^2} + \frac{1}{x^4}}{1 + \frac{1}{x^4}}$$
$$= \frac{1 - 0 + 0}{1 + 0}$$
$$= 1$$

3. (2 points) Given that

 $f(x) = \sin x - \cos x \qquad \qquad f'(x) = \cos x + \sin x \qquad \qquad f''(x) = -\sin x + \cos x$

Find the inflection points of f on the interval $[0, 2\pi]$.

Solution: The potential inflection points are where f''(x) = 0 which is when $\sin x = \cos x$ which occurs at $\pi/4$ and $5\pi/4$ on the interval $[0, 2\pi]$. We still need to check that f''(x) changes signs at these points. So plug in points in the intervals $(0, \pi/4)$, $(\pi/4, 5\pi/4)$, and $(5\pi/4, 2\pi)$, and verify that f'' does in fact change sign at $x = \pi/4$ and $x = 5\pi/4$.