Name: ______________________________ **Section**: ______________

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (2 points) **Multiple Choice. Circle the best answer. No partial credit available**

   Evaluate the limit $\lim_{t\to 4} \dfrac{\sin(\sqrt{t}-2)}{t-4}$

   A. 4

   B. 2

   C. 1/2

   D. 1/4

   E. None of the above.

Solution: Since $t-4 = (\sqrt{t}-2)(\sqrt{t}+2)$, we can re-write $\frac{\sin(\sqrt{t}-2)}{t-4}$ as $\frac{\sin(\sqrt{t}-2)}{\sqrt{t}-2} \cdot \frac{1}{\sqrt{t}+2}$. Now, taking the limit, we get

$$\lim_{t\to 4} \frac{\sin(\sqrt{t}-2)}{t-4} = \left(\lim_{t\to 4} \frac{\sin(\sqrt{t}-2)}{\sqrt{t}-2}\right)\left(\lim_{t\to 4} \frac{1}{\sqrt{t}+2}\right) = (1)\left(\frac{1}{\sqrt{4}+2}\right) = \frac{1}{4}$$

2. (2 points) Write the equation for the tangent line to the graph of $f(x) = \sqrt{x}$ at $x = 4$.

Solution: We know the equation for the tangent line at $x = c$ is:

$$y = f(c) + f'(c)(x-c)$$

Since we want the tangent line at 4, we calculate using $c = 4$:

$$y = f(4) + f'(4)(x-4)$$
$$y = \sqrt{4} + \frac{1}{2\sqrt{4}}(x-4)$$
$$y = 2 + \frac{1}{4}(x-4)$$

If you prefer slope-intercept form, this can be re-written as $y = \frac{x}{4} + 1$.

Continue on to back side

3. Consider $f(x) = \begin{cases} 2x - x^2 & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$.

   (a) (1 point) Is $f(x)$ continuous at $x = 1$? Show work to support your answer!

   Solution:

$$\lim_{x \to 1^-} f(x) \stackrel{?}{=} \lim_{x \to 1^+} f(x)$$
$$\lim_{x \to 1^-} [2x - x^2] \stackrel{?}{=} \lim_{x \to 1^+} [x^2]$$
$$1 = 1$$

   So yes the left and right limits are both equal to 1. Also note $f(1) = 1$ giving us $\boxed{\text{Yes. } f \text{ is continuous at } 1}$.

   (b) (1 point) Is $f(x)$ differentiable at $x = 1$? Show work to support your answer!

   Solution: In order to be differentiable, we must have that

$$\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h}$$

   When $x < 1$, $f(x) = 2x - x^2$, and so the left-hand limit will just be the derivative of $2x - x^2$ evaluated at $x = 1$. The derivative of $2x - x^2$ is just $2 - 2x$, and so we get that

$$\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h} = 2 - 2(1) = 0$$

   On the other hand, when $x \geq 1$, $f(x) = x^2$, and so the right-hand limit will be the derivative of x^2 evaluated at $x = 1$. The derivative of x^2 is $2x$, so we get

$$\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = 2(1) = 2$$

   So $\boxed{\text{No. } f(x) \text{ is not differentiable at } 1}$, because $\frac{d}{dx}\Big|_{x=1} (x^2) \neq \frac{d}{dx}\Big|_{x=1} (2x - x^2)$.