Name: ______________________ **Card #**: ______________

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (1 point) **Fill-in-the-Blank. No partial credit available**

   Find all numbers c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = \sqrt{x} - \frac{1}{3}x$ on the interval $[0, 9]$.

   $c =$ 9/4 ______________

Solution: The average rate of change over the interval $[0, 9]$ is given by

$$\frac{f(9) - f(0)}{9 - 0} = \frac{0 - 0}{9} = 0$$

So the **Mean Value Theorem** says that there is a number c between 0 and 9 so that $f'(c) = 0$. Taking the derivative, we get

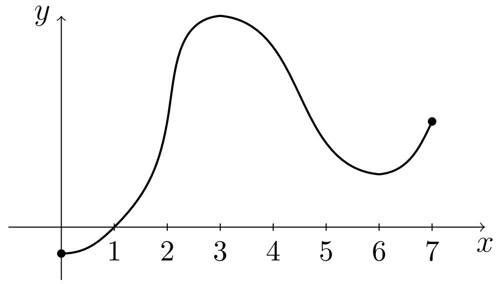
$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{3}$$

Since $f'(c) = 0$, we can solve algebraically:

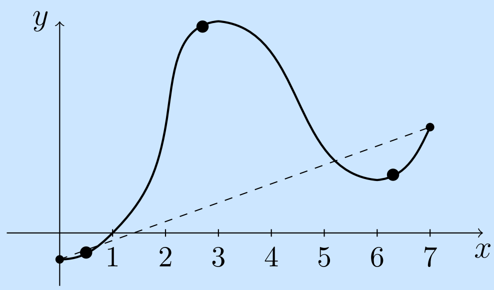
$$\begin{aligned}
f'(c) &= 0 \\
\frac{1}{2\sqrt{c}} - \frac{1}{3} &= 0 \\
\frac{1}{2\sqrt{c}} &= \frac{1}{3} \\
2\sqrt{x} &= 3 \\
\sqrt{x} &= \frac{3}{2} \\
x &= \left(\frac{3}{2}\right)^2 \\
x &= \frac{9}{4}
\end{aligned}$$

Continue on to back side

2. (1 point) Consider the graph of $f(x)$ to the right. Can we apply the Mean Value Theorem to this function on the interval $[0, 7]$? If yes, estimate the values of c that satisfy the conclusion of the Mean Value Theorem. If no, explain why we can't.



Solution: The function appears to be continuous and differentiable on $[0, 7]$ so we can apply the mean value theorem. The values of c will be where the tangent line is paralell to the secant line over the interval $[0, 7]$. It appears that possible c values include: 0.5, 2.5, and 6.5.



3. (2 points) Find the intervals on which $f(x) = \dfrac{x}{x^2+1}$ is increasing or decreasing.

Solution:

$$f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

The derivative is positive if $x^2 < 1$, which happens when x is between -1 and 1. The derivative is negative when $x^2 > 1$, which happens when either $x < -1$ or $x > 1$. So we conclude that f is increasing on $(-1, 1)$ and decreasing on $(-\infty, -1) \cup (1, \infty)$.