Name: ______________________________ **Card #**: ______________

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (1 point) **Multiple Choice. Circle the best answer. No partial credit available**

   Find the derivative of $f(t) = \dfrac{\tan t - 1}{\sec t}$ at $t = \pi$.

   A. $f'(\pi) = -1$

   B. $f'(\pi) = 0$

   C. $f'(\pi) = 1$

   D. $f'(\pi) = 2$

   E. None of the above.

Solution: You can use the quotient rule, using the derivatives of $\tan(t)$ and $\sec(t)$:

$$f'(t) = \frac{\sec^2(t) \cdot \sec(t) - (\tan(t) - 1)\sec(t)\tan(t)}{\sec^2(t)} = \frac{\sec^3(t) - (\tan(t) - 1)\sec(t)\tan(t)}{\sec^2(t)}$$

Then just plug in $t = \pi$ to get

$$f'(\pi) = \frac{(-1)^3 - (0-1)(-1)(0)}{(-1)^2} = -1$$

An easier way however, is to first simplify the expression by writing $\tan(t)$ as $\frac{\sin(t)}{\cos(t)}$ and $\sec(t)$ as $\frac{1}{\cos(t)}$, so we can also write $f(t)$ as

$$f(t) = \sin(t) - \cos(t)$$

Then differentiating, we get

$$f'(t) = \cos(t) + \sin(t)$$

Plugging in $t = \pi$, we get

$$f'(\pi) = -1 + 0 = -1$$

Continue on to back side

2. (1 point) **Fill-in-the-Blank. No partial credit available**

Suppose f and g are functions of x that are differentiable at $x = 1$ and that

$$f(1) = 7 \qquad f'(1) = -5 \qquad g(1) = -4 \qquad g'(1) = 2$$

(a) $\dfrac{d}{dx}(fg)\Big|_{x=1} = \underline{f'(1) \cdot g(1) + f(1) \cdot g'(1) = 34}$

(b) $\dfrac{d}{dx}\left(\dfrac{f}{g}\right)\Big|_{x=1} = \underline{\dfrac{f'(1) \cdot g(1) - f(1) \cdot g'(1)}{g(1)^2} = 3/8}$

(c) $\dfrac{d}{dx}(2g - 3f)\Big|_{x=1} = \underline{2 \cdot g'(1) - 3 \cdot f'(1) = 19}$

Extra Work Space.

3. (1 point) Find the derivative of $f(x) = \sin\left(\sqrt{\frac{1}{x+2}}\right)$

Solution: If we denote $g(x) = \sin(x)$, $h(x) = \sqrt{x}$, and $k(x) = \frac{1}{x+2}$, then we can write f as the composition $f = g \circ h \circ k$. Then the chain rule says:

$$f' = g'(h(k)) \cdot h'(k) \cdot k'$$

We know the derivatives of g, h, and k:

$$g' = \cos(x)$$

$$h' = \frac{1}{2\sqrt{x}}$$

$$k' = \frac{-1}{(x+2)^2}$$

So we have that

$$f' = \cos\left(\sqrt{\frac{1}{x+2}}\right) \cdot \frac{1}{2\sqrt{\frac{1}{x+2}}} \cdot \frac{-1}{(x+2)^2}$$

4. (1 point) Use the quotient rule (and the fact that $\cot(x) = \frac{\cos(x)}{\sin(x)}$) to show that

$$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

Solution:

$$\begin{aligned}
\frac{d}{dx}\left(\cot(x)\right) &= \frac{d}{dx}\left(\frac{\cos(x)}{\sin(x)}\right) \\
&= \frac{\frac{d}{dx}(\cos(x)) \cdot \sin(x) - \cos(x) \cdot \frac{d}{dx}(\sin(x))}{\sin(x)^2} \\
&= \frac{-\sin(x)^2 - \cos(x)^2}{\sin(x)^2} \\
&= \frac{-(\sin(x)^2 + \cos(x)^2)}{\sin(x)^2} \\
&= \frac{-1}{\sin(x)^2} \\
&= -\csc(x)^2
\end{aligned}$$