

Name: _____ Card #: _____

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**

If you have a question raise your hand and I will come to you.

1. **Fill-in-the-Blank. No partial credit available**

Solve the initial value problem:

$$f''(t) = 10 \cos(t) - \sin(t), \quad f'(0) = -1, \quad f(\pi) = 1$$

(a) (2 points) $f'(t) =$ $10 \sin t + \cos t - 2$

(b) (2 points) $f(t) =$ $-10 \cos t + \sin t - 2t + (2\pi - 9)$

2. (2 points) Estimate the area under the graph of $f(x) = 7 + x - x^2$ on the interval $[-3, 3]$, using a **right-hand** sum with $n = 3$ sub-intervals.

Solution: Since the interval $[-3, 3]$ has width 6, we know that $\Delta x = \frac{6}{3} = 2$. So our sub-intervals are $[-3, -1]$, $[-1, 1]$, and $[1, 3]$. For a right-hand sum, we get

$$\Delta x (f(-1) + f(1) + f(3)) = 2(5 + 7 + 1) = 26$$

3. I am driving my car when all of a sudden a tree falls into the road 50 feet in front of me. I am currently driving 60 ft/s ($\approx 41mph$) when I slam on my breaks so that my velocity is given by the function $v(t) = 60 - 7t - t^2 = (5 - t)(12 + t)$.

(a) (1 point) How many seconds did it take me to come to a complete stop?

Solution: The velocity is zero when $t = 5$.

(b) (3 points) Did I hit the tree? (Justify your answer)

Solution: The position function is the antiderivative of the velocity, and so it is given by

$$p(t) = 60t - \frac{7}{2}t^2 - \frac{1}{3}t^3 + C$$

Let's just assume that $C = 0$ (assume that at time $t = 0$, we are at position 0). We decided in part (a) that it would take 5 seconds to stop. We just need to see how far we would go in 5 seconds (is it more or less than 50 feet?). We see that

$$p(5) = 60(5) - \frac{7 \cdot 25}{2} - \frac{125}{3} = \frac{1025}{6} > 50$$

So the driver *does* hit the tree.