Example 2.7 Evaluate the following sums

(a) $\displaystyle\sum_{i=-1}^{3} 2^{3-i}$

Solution: $\displaystyle\sum_{i=-1}^{3} 2^{3-i} = 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 16 + 8 + 4 + 2 + 1 = 31$

(b) $\displaystyle\sum_{i=0}^{4} (1-i)(2+i)$

Solution: $\displaystyle\sum_{i=0}^{4} (1-i)(2+i) = (1)(2) + (0)(3) + (-1)(4) + (-2)(5) + (-3)(6) = 2 - 4 - 10 - 18 = -30$

Example 2.8 Write the sum: $\sqrt{3} - \sqrt{5} + \sqrt{7} - \sqrt{9} + \cdots + \sqrt{27}$ in sigma notation.

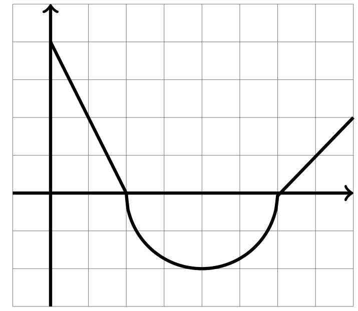
Solution: $\displaystyle\sum_{i=1}^{13} (-1)^{i-1}\sqrt{2k+1}$

Example 2.9 Evaluate the integral $\displaystyle\int_{-1}^{4} (x^2 - x + 1)dx$.

Solution: Now that we know the **Fundamental Theorem of Calculus**, we can compute it easily:

$$\begin{aligned}
\int_{-1}^{4} (x^2 - x + 1)dx &= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_{-1}^{4} \\
&= \left(\frac{64}{3} - \frac{16}{2} + 4\right) - \left(\frac{-1}{3} - \frac{1}{2} - 1\right) \\
&= \frac{115}{6}
\end{aligned}$$

Example 2.10 Evaluate the definite integral $\int_1^7 f(x)\ dx$.
Where $f(x)$ is given by the function to the right.



Solution: We can see from the picture that $\int_1^2 f(x)dx$ is given by the area of a triangle with base 1 and height 2, and so the area is 1. Similarly, $\int_6^7 f(x)dx$ is the area of a triangle of base 1 and height 1, and so the area is $\frac{1}{2}$. Finally, the integral $\int_2^6 f(x)dx$ is (negative of) half the area of a circle with radius 2, and so the area is 2π. So all together, we have

$$\int_1^7 f(x)dx = \int_1^2 f(x)dx + \int_2^6 f(x)dx + \int_6^7 f(x)dx = 1 + 2\pi + \frac{1}{2} = \frac{3}{2} + 2\pi$$

Example 2.11 Prove that $\int_a^b x^2 dx = \frac{b^3 - a^3}{3}$.

Solution: This is obvious using the **Fundamental Theorem of Calculus**, since $\int x^2 dx = \frac{x^3}{3}$.