Appendix E - Sigma Notation

Definition(s) 0.1. If $a_m, a_{m+1}, \ldots, a_{n-1}, a_n$ are real numbers and m and n are integers such that $m \leq n$, then

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \cdots + a_{n-1} + a_n$$

Definition(s) 0.2. This way of short-handing sums of many numbers is called **sigma notation** (uses the Greek letter Σ "Sigma"). The letter i above is called the **index of summation** and it takes on consecutive integer values starting with m and ending with n.

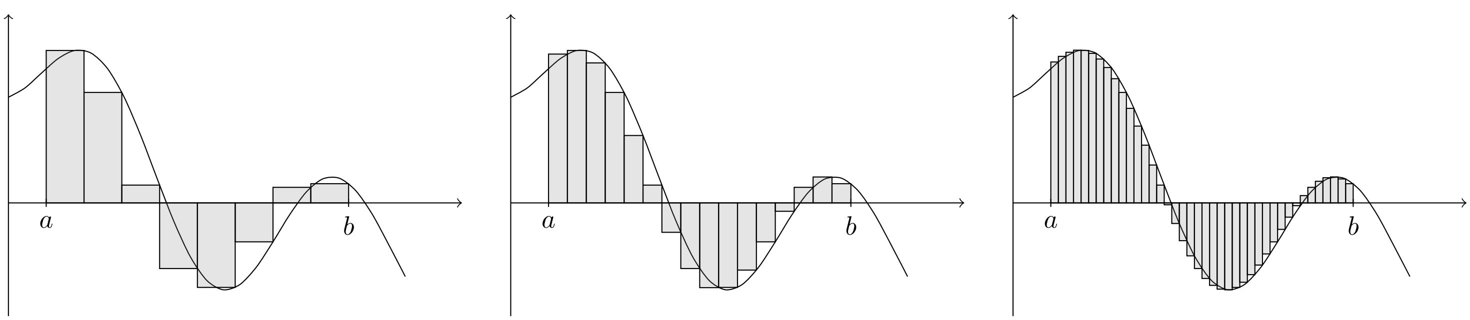
Theorem 0.3. If c is any constant then:

1. $\displaystyle\sum_{i=m}^{n} ca_i = c\sum_{i=m}^{n} a_i$
2. $\displaystyle\sum_{i=m}^{n} (a_i + b_i) = \sum_{i=m}^{n} a_i + \sum_{i=m}^{n} b_i$
3. $\displaystyle\sum_{i=m}^{n} (a_i - b_i) = \sum_{i=m}^{n} a_i - \sum_{i=m}^{n} b_i$

Theorem 0.4. Let c be a constant and n a positive integer. Then

1. $\displaystyle\sum_{i=1}^{n} 1 = n$
2. $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
3. $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

2 The Definite Integral



Theorem 2.1 (Definite Integral)**.** If f is integrable on $[a, b]$, then

$$\int_a^b f(x)\ dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where

$$\Delta x = \frac{b-a}{n} \qquad \text{and} \qquad x_i = a + i\Delta x$$

Remark 2.2. The definite integral, $\displaystyle\int_a^b f(x)\ dx$ gives the **net area** between the curve f and the $x-$ axis.

Example 2.3 (Instructor)**.** Evaluate the following sums

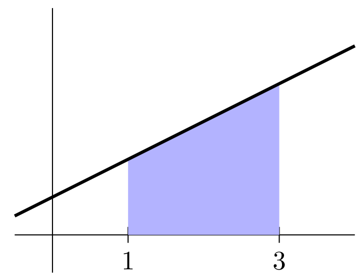
1. $\displaystyle\sum_{i=0}^{4} \frac{2k-1}{2k+1}$

2. $\displaystyle\sum_{i=0}^{4} (2-3i)$

3. $\displaystyle\sum_{i=1}^{38} (3^i - 3^{i-1})$

Example 2.4 (Instructor)**.** Write th sum: $\sqrt{3} + \sqrt{4} + \cdots + \sqrt{25}$ in sigma notation

Example 2.5 (Instructor)**.** Use the definition of the definite integral to verify that $\displaystyle\int_1^3 \left(\frac{x+1}{2}\right)\, dx = 3$



Example 2.6 (Instructor)**.** Evaluate the definite integral $\displaystyle\int_1^3 \left(x^2+1\right)\, dx$

Example 2.7 (Student)**.** Evaluate the following sums

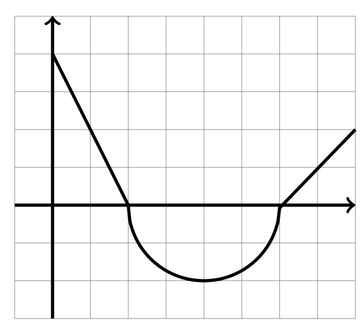
1. $\displaystyle\sum_{i=-1}^{3} 2^{3-i}$

2. $\displaystyle\sum_{i=0}^{4} (1-i)(2+i)$

Example 2.8 (Student)**.** Write th sum: $\sqrt{3} - \sqrt{5} + \sqrt{7} - \sqrt{9} + \cdots + \sqrt{27}$ in sigma notation

Example 2.9 (Student)**.** Evaluate the definite integral $\displaystyle\int_{-1}^4 \left(x^2 - x + 1\right)\, dx$

Example 2.10 (Student)**.** Evaluate the definite integral $\displaystyle\int_1^7 f(x)\, dx$. Where $f(x)$ is given by the function to the right.



Example 2.11 (Student, **Bonus:** as time allows)**.** Prove that $\displaystyle\int_a^b x^2\, dx = \frac{b^3-a^3}{3}$