8 Newton's Method

Remark 8.1. Newton's method is an extremely powerful technique used in finding roots of equations in general the convergence is quadratic: as the method converges on the root, the difference between the root and the approximation is squared (the number of accurate digits roughly doubles) at each step. However, there are some difficulties with the method which we will discuss below. Newton's method can be altered to help approximate solutions to many equations.

Pictures of the idea of Newton's Method

Theorem 8.2 (Newton's Method)**.** If x_1 is the initial guess of some root of $f(x)$ then

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Remark 8.3. Newton's method may fail to converge to an answer or may find the wrong root. See the pictures below for how this can happen. When this occurs it can usually be fixed by selecting an alternative initial guess (x_1).

Horizontal tangent	**Never converges**	**Converges to wrong root**

9 Antiderivatives

Definition(s) 9.1. A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Theorem 9.2. If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Example 9.3 (Instructor)**.** Approximate the root of $f(x) = x^3 + x + 4$ by taking the initial guess $x_1 = -1$. Compute x_2 and x_3. *Don't simplify* x_3.

Example 9.4 (Instructor)**.** Use Newton's method to approximate the positive root of $x^2 = \sin x$. Take $x_1 = \pi/3$. Calculate x_2.

Example 9.5 (Instructor)**.** Approximate $\sqrt{5}$ using

(a) Linearization with $a = 4$

(b) Newton's method with $x_1 = 2$. Compute x_2 and x_3.

Example 9.6 (Instructor)**.** Find the most general antiderivative of the functions below.

(a) $f(x) = \pi + \dfrac{1}{x}$

(b) $f(x) = \sqrt{x}(6 + 7x)$

Example 9.7 (Instructor)**.** A particle is moving with velocity $v(t) = \sin t - \cos t$, and has initial position $s(0) = 0$. Find the position function of the particle.

Example 9.8 (Student)**.** Use Newton's method to approximate all roots of the equation $\dfrac{1}{x} = 1 + x^3$. Sketch these graphs and use the IVT to select to best integer values for starting points. Stop at x_2.

Example 9.9 (Student)**.** Use Newton's method to approximate $\sqrt[3]{29}$. Calculate up to x_3. Do not simplify.

Example 9.10 (Student)**.** Find the most general antiderivative of the functions below.

(a) $f(x) = (3 + 2x)^2$

(b) $f(x) = \dfrac{3 - x + 5x^3}{x}$

Example 9.11 (Student)**.** Below is a graph of f'. Sketch the graph of f if f is continuous and $f(0) = -2$