5 Curve Sketching

Remark 5.1 (General Guidelines for Curve Sketching)**.**
When trying to sketch the graph of a given function $f(x)$, ask yourself the following:

1. What is the domain of f?
2. What are the x and y intercepts of the graph?
   (a) Evaluate $f(0)$ for y-intercept
   (b) Solve $f(x) = 0$ for x-intercept
3. Does the graph have any symmetry (is f an odd function? an even function? neither?)
   (a) Even if $f(-x) = f(x)$ for all x
   (b) Odd if $f(-x) = -f(x)$ for all x
4. Does the graph have any asymptotes (horizontal, vertical, slant)?
   (a) Has horizontal asymptote $y = c$ if $\lim_{x\to\infty} f(x) = c$ or if $\lim_{x\to-\infty} f(x) = c$.
   (b) Has vertical asymptote $x = c$ if either of $\lim_{x\to c^+} f(x)$ or $\lim_{x\to c^-} f(x)$ are equal to either ∞ or $-\infty$.
   (c) Has slant asymptote $y = mx + b$ if $\lim_{x\to\infty} (f(x) - (mx + b)) = 0$ or if $\lim_{x\to-\infty} (f(x) - (mx + b)) = 0$.
5. Where is $f(x)$ increasing and decreasing?
   (a) See **Theorem 3.1**
6. Where are the local maximum and minimum points?
   (a) Use **First Derivative Test** or **Second Derivative Test**
7. Where is the graph concave up and where is it concave down?
   (a) See **Theorem 3.12**
8. Where are the inflection points?
   (a) Find where $f''(x)$ changes sign

Example 5.2 (Instructor)**.**

Sketch graphs of the following:

(a) $y = x(x-4)^3$

(b) $y = \dfrac{x^3}{x^2+1}$

(c) $y = \dfrac{x}{\sqrt{x^2+1}}$

(d) $y = \sin^3(x) = (\sin(x))^3$

Example 5.3 (Student)**.**

Sketch graphs of the following:

(a) $y = x^5 - 5x$

(b) $y = \dfrac{x}{x-1}$

(c) $y = 1 + \dfrac{1}{x} + \dfrac{1}{x^2}$

(d) $y = x + \cos(x)$