3 (b) - Derivatives and Graphs

Definition(s) 3.11. If the graph of f lies above all of its tangents on an interval I, then it is called **concave up** on I. If the graph of f lies below all of its tangents on I, it is called **concave down** on I.

Picture:

Theorem 3.12 (Concavity Test)**.**

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Definition(s) 3.13. A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

Theorem 3.14. Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

4 Limits at Infinity; Horizontal Asymptotes

Definition(s) 4.1. Let f be a function defined on some interval (a, ∞). Then $\lim_{x\to\infty} f(x) = L$ means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

Definition(s) 4.2. Let f be a function defined on some interval $(-\infty, a)$. Then $\lim_{x\to-\infty} f(x) = L$ means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently negative.

Definition(s) 4.3. The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

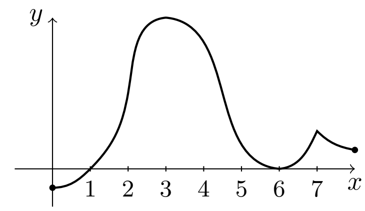
$$\lim_{x\to-\infty} f(x) = L \qquad \text{or} \qquad \lim_{x\to\infty} f(x) = L$$

Theorem 4.4. If $r > 0$ is a rational number, then

$$\lim_{x\to\infty} \frac{1}{x^r} = 0$$

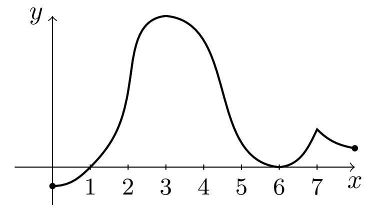
If $r > 0$ is a rational number such that x^r is defined for all x, then

$$\lim_{x\to-\infty} \frac{1}{x^r} = 0$$



Example 4.5 (Instructor).

Suppose that the graph to the right is of is of f.
For which values of x does f have an inflection point?



Example 4.6 (Instructor).

Suppose that the graph to the right is of is of f'.
For which values of x does f have an inflection point?

Example 4.7 (Instructor).

Find where $f(x) = x\sqrt{6-x}$ is concave up and where it is concave down. Where are the inflection points?

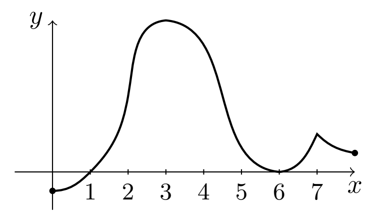
Example 4.8 (Instructor).

Find the limits or show that they do not exist

(a) $\lim_{x\to\infty} \frac{2x-1}{5x+3}$

(b) $\lim_{x\to\infty} x \sin\frac{1}{x}$

(c) $\lim_{x\to\infty} \frac{\sqrt{x}-x^2}{2x+x^2}$



Example 4.9 (Student).

Suppose that the graph to the right is of is of f''.
For which values of x does f have an inflection point?

Example 4.10 (Student).

Find where $f(x) = x - 4\sqrt{x}$ is concave up and where it is concave down. Where are the inflection points?

Example 4.11 (Student).

Use the second derivative test to classify the critical points for $f(x) = \frac{x^2}{x-1}$

Example 4.12 (Student).

Find the limits or show that they do not exist

(a) $\lim_{x\to\infty} \frac{(2x+1)^2}{(x-1)^2(x^2+x)}$

(b) $\lim_{x\to-\infty} \frac{x^2}{\sqrt{3x^4+1}}$

(c) $\lim_{x\to\infty} \sqrt{x^2+1}$

Example 4.13 (Student).

Find the horizontal asymptotes of $f(x) = \frac{4x^3+6x^2-2}{5x^3-x+1}$