1 Maximum and Minimum Values

Definition(s) 1.1. Let c be a number in the domain D of a function f. Then $f(c)$ is the

- **absolute (global) maximum** value of f on D if $f(c) \geq f(x)$ for all x in D.
- **absolute (global) minimum** value of f on D if $f(c) \leq f(x)$ for all x in D.
- **local maximum** value of f on D if $f(c) \geq f(x)$ for all x near c.
- **local maximum** value of f on D if $f(c) \leq f(x)$ for all x near c.

Maximums and minimums are often referred to as **extreme values**.
Pictures:

Remark 1.2. The book uses "**near** c" to mean technically that the statement is true in some **open** interval containing c. Sometimes these definitions can make you crazy. Look at **Example 1.10 below**.

Theorem 1.3. If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Definition(s) 1.4. A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Remark 1.5. If f has a local maximum or minimum at c, then c is a critical number of f.

Theorem 1.6. To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints (a and b).
3. The largest of the values from above is the absolute maximum value; the smallest is the absolute minimum value.

Remark 1.7. In **Section 3.3** we will find a way to classify local maximums and minimums on any domain! (not just closed intervals)

Example 1.8 (Instructor).

Find the absolute maximum an minimum values of $f(x) = x^3 - 12x + 1$ on the interval $[1, 4]$

Example 1.9 (Instructor).

Sketch a graph of a function f that is continuous on $[1, 5]$ and has all of the following properties

- An absolute minimum at 2
- An absolute maximum at 3
- A local minimum at 4

Example 1.10 (Instructor).

Sketch a graph of a function f who has domain $[-2, 4]$ that has an absolute maximum but no local maximum.

Example 1.11 (Instructor).

Find the critical numbers for the function $f(x) = \dfrac{x-1}{x^2+4}$

Example 1.12 (Student).

Find the critical numbers for the function $f(x) = \sqrt{1-x^2}$

Example 1.13 (Student).

Sketch a graph of a function f that has two local maxima, one local minimum, and no absolute minimum.

Example 1.14 (Student).

Find the absolute maximum an minimum values of $f(x) = 12 + 4x - x^2$ on the interval $[0, 5]$

Example 1.15 (Student).

Find the absolute maximum an minimum values of $f(x) = \dfrac{x}{x^2 - x + 1}$ on the interval $[0, 3]$

Example 1.16 (Student).

Find the absolute maximum an minimum values of $f(x) = 2\cos t + \sin(2t)$ on the interval $[0, \pi/2]$