9 Linear Approximations and Differentials

Remark 9.1. **Linear approximation** and **tangent line approximation** are two names for using the equation of a tangent line to approximate a function

Definition(s) 9.2.

$$L(x) = f(x) + f'(a)(x - a)$$

is called the **linearization** of f at a.

Note: compare this to the equation of a tangent line through $(a, f(a))$: $y - f(a) = f'(a)(x - a)$.

Remark 9.3. An equivalent notion to linearization is differentials. Consider the definitions:

$dx = x - a$ $dy = y - f(a)$ (Note y and x are from the tangent line, not the function f.)

Then we can plug these into our tangent line equation to get

$$dy = f'(a)dx$$

More generally it can be written as

$$dy = f'(x)dx$$

Where dy and dx are considered variables in their own right.

Definition(s) 9.4.

$$\Delta x = x - a \qquad \Delta y = f(x + \Delta x) - f(x)$$

Example 9.5 (Instructor)**.**

(a) Find the linearization $L(x)$ of the function $f(x) = \sqrt{x}$ at $a = 9$

(b) Use the linearization to approximate $\sqrt{10}$

Example 9.6 (Instructor)**.**

(a) Find the differential dy of $y = \cos(\pi x)$

(b) Evaluate dy for $x = 1/3$ and $dx = -0.02$

Example 9.7 (Instructor)**.**

Use linear approximation to estimate $\sqrt[3]{1001}$

Example 9.8 (Student)**.**

(a) Find the linearization $L(x)$ of the function $f(x) = \sin x$ at $a = \pi/4$

(b) Use the linearization to approximate $\sin(11\pi/40)$

Example 9.9 (Student)**.**

(a) Find the differential dy of $y = \sqrt{x^2 + 8}$

(b) Evaluate dy for $x = 1$ and $dx = 0.02$

Example 9.10 (Student)**.**

Use linear approximation to estimate $\dfrac{1}{4.002}$