8 (b) - Related Rates

Remark 8.1 (General Technique for solving Related Rate Problems)**.**

(i) Relating given quantities with an equation.

(ii) Differentiating the equation (using implicit differentiation and the chain rule).

(iii) Solving for some rate of change (derivative).

Remark 8.2.

Geometric/Trig formulas you are expected to know

- Circles

$$C = \pi d \qquad d = 2r \qquad A = \pi r^2$$

- Triangles

$$A = \frac{1}{2}bh \qquad c^2 = a^2 + b^2 \text{ (for right } \Delta\text{s)}$$

$$\sin\theta = \frac{O}{H} \text{ (for right } \Delta\text{s)} \qquad \cos\theta = \frac{A}{H} \text{ (for right } \Delta\text{s)} \qquad \tan\theta = \frac{O}{A} = \frac{\sin\theta}{\cos\theta} \text{ (for right } \Delta\text{s)}$$

- Rectangles (Squares)

$$P = 2l + 2w \qquad A = lw$$

- Spheres

$$V = \frac{4}{3}\pi r^3 \qquad SA = 4\pi r^2$$

- Cones

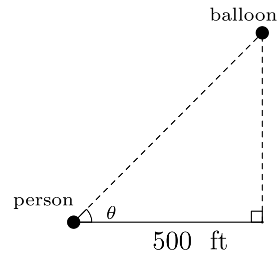
$$V = \frac{h}{3}(\text{area of base})$$

- Cylinders

$$V = h(\text{area of base})$$

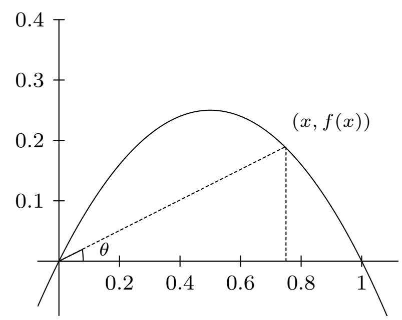
Example 8.10 (Instructor).

A balloon is rising vertically above a field, and a person 500 feet away from the spot on the ground underneath the balloon is watching it, measuring the angle of inclination, θ. When the angle is $\pi/4$ radians, the angle is increasing at $\frac{1}{10}$ radians per minute. At that moment, how fast is the balloon rising?



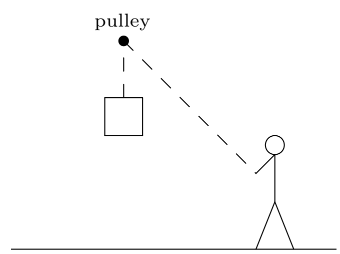
Example 8.11 (Student).

Let $f(x) = x - x^2 = x(1-x)$, and let θ be the angle between the positive x-axis and the line joining the point $(x, f(x))$ with the origin. At what rate is θ changing, with respect to x, when $x = 1 - \frac{1}{\sqrt{3}}$?



Example 8.12 (Student).

A person is lifting a weight with a pulley. The pulley is 25 feet off the ground, the rope is 100 feet long, and the person is holding the end of the rope 5 feet off the ground. If the person is walking backwards (away from the pulley) at a constant rate of 5 ft/sec, how fast is the weight rising when the person is 10 feet from the spot on the ground directly under the pulley?



Example 8.13 (Student).

A spotlight on the ground shines on a wall 12 meters away. If a man 2 meters tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?