7 Rates of Change in the Natural and Social Sciences

Definition(s) 7.1.
The **instantaneous rate of change of** $y = f(x)$ **with respect to** x is the slope of the tangent line (a.k.a. derivative). Using Leibniz notation, we write:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

As was brought up in Section 2.1, the units for dy/dx are the units for y divided by the units for x.

Remark 7.2 (From Section 2.2)**.**

If $s = f(t)$ is the position function of an object that is moving in a straight line, then $v(t) = s'(t)$ represents the **velocity** at time t. Also, $a(t) = v'(t) = s''(t)$ is the **acceleration** of the object at time t.

Remark 7.3.

Some common phrases and their mathematical interpretations

- When is the object at rest (or stand still)?
- When is the object moving forward/backward?
- When is the speeding up/down?
- Find the total distance traveled
- (Gravity Problems) When does an object achieve its maximum height?

6 (a) - Implicit Differentiation

Definition(s) 6.1.
Implicit Differentiation is a method of differentiating both sides of an equation with respect to x and then solving the resulting equation for y'.

Remark 6.2.

Be careful that you are applying power, product, quotient, and chain rules correctly.

Remark 6.3.

Khan Academy has 7 videos and practice problems all about implicit differentiation. Feel free to check them out at:

`https://www.khanacademy.org/math/differential-calculus/taking-derivatives`

Example 6.4 (Instructor).

If a ball is thrown vertically upward with a velocity of 80ft/s, then its height after t seconds is $s = 80t - 16t^2$.

(a) What is the maximum height reached by the ball?

(b) What is the velocity of the ball when it is 96 feet above the ground on its way up? On its way down?

Example 6.5 (Instructor).

A particle moves according to the law of motion $s(t) = \cos(\pi t/4)$ with $0 \le t \le 10$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t.

(b) What is the velocity after 3 seconds?

(c) When is the particle at rest?

(d) When is the particle moving in the positive direction?

(e) Find the total distance traveled in the first 8 seconds?

(f) When is the particle speeding up? When is it slowing down?

Example 6.6 (Instructor).

Find dy/dx of $x^3 + y^3 = 1$ using implicit differentiation.

Example 6.7 (Instructor).

Use implicit differentiation to find an equation of the tangent line to the curve $y \sin 2x = x \cos 2y$ at the point $(\pi/2, \pi/4)$.

Example 6.8 (Student).

The height (in meters) of a projectile shot vertically upward from a point 2 m above ground level with an initial velocity of 25 m/s is $h = 2 + 25t - 5t^2$ after t seconds.

(a) Find the velocity after 2 seconds.

(b) When does the projectile reach its maximum height?

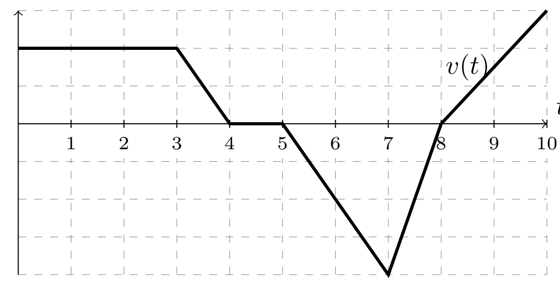
(c) What is the maximum height?

(d) When does it hit the ground?

(e) With what velocity does it hit the ground?

Example 6.9 (Student, FS14 E1).

The figure below shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for t in a closed interval $[0, 10]$.



Fill in the following blanks. No partial credit available. No work needed. Use interval notation where appropriate.

(a) The particle is moving forward during: __________

(b) The particle's speed is increasing during: __________

(c) The particle has positive acceleration during: __________

(d) The particle has zero acceleration during: __________

(e) The particle achieves its greatest speed at: __________

(f) The particle stands still for more than an instant during: __________

Example 6.10 (Student).

Find dy/dx by implicit differentiation of $\tan(x - y) = \dfrac{y}{1 + x^2}$.

Example 6.11 (Student Ex2 in book).

(a) Find y' if $x^3 + y^3 = 6xy$.

(b) Find and equation of the tangent line at the point $(3, 3)$.

(c) At what point in the first quadrant is the tangent line horizontal?