6 (b) - Implicit Differentiation

Remark 6.12.
Today we will practice implicit differentiation with 3 variables: x, y, and t. Where $x = x(t)$ and $y = y(t)$, that is x and y depend on t. This will lead us nicely into related rates problems.

8 (a) - Related Rates

Remark 8.1 (General Technique for solving Related Rate Problems)**.**

(i) Relating given quantities with an equation.

(ii) Differentiating the equation (using implicit differentiation and the chain rule).

(iii) Solving for some rate of change (derivative).

Remark 8.2.

Geometric/Trig formulas you are expected to know

- Circles

$$C = \pi d \qquad d = 2r \qquad A = \pi r^2$$

- Triangles

$$A = \frac{1}{2}bh \qquad c^2 = a^2 + b^2 \text{ (for right } \Delta\text{s)}$$

$$\sin\theta = \frac{O}{H} \text{ (for right } \Delta\text{s)} \qquad \cos\theta = \frac{A}{H} \text{ (for right } \Delta\text{s)} \qquad \tan\theta = \frac{O}{A} = \frac{\sin\theta}{\cos\theta} \text{ (for right } \Delta\text{s)}$$

- Rectangles (Squares)

$$P = 2l + 2w \qquad A = lw$$

- Spheres

$$V = \frac{4}{3}\pi r^3 \qquad SA = 4\pi r^2$$

- Cones

$$V = \frac{h}{3}(\text{area of base})$$

- Cylinders

$$V = h(\text{area of base})$$

Example 8.3 (Instructor).

Suppose you have the following information:

$$y(t) = \frac{\pi}{12}x(t)^3 \qquad y'(t) = 9 \qquad y(a) = 6$$

Find $y'(a)$.

Example 8.4 (Instructor).

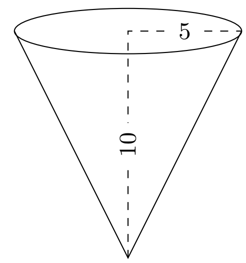
Suppose you have the following information:

$$169 = x(t)^2 + y(t)^2 \qquad y'(t) = -3 \qquad x(a) = 5 \qquad x(t) \geq 0 \qquad y(t) \geq 0$$

Find $x'(a)$.

Example 8.5 (Instructor).

A cone-shaped tank is filling with water at a constant rate of 9 ft^3/min. The tank is 10 ft tall, and has a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



Example 8.6 (Instructor).

A 13 foot ladder is leaning against a wall, and is sliding down, with the top of the ladder moving downwards along the wall at 3 ft/sec. How fast is the bottom of the ladder moving away from the wall when the bottom is 5 ft from the base of the wall?

Example 8.7 (Student).

A sphere is growing, its volume increasing at a constant rate of 10 in^3 per second. Let $r(t)$, $V(t)$, and $S(t)$, be the radius, volume, and surface area of the sphere at time t. If $r(1) = 2$, then compute:

(a) $r'(1)$

(b) $S'(1)$

Example 8.8 (Student).

A circle is growing, its radius (in inches) given by $r(t) = \sqrt{t}$ for t in seconds. How fast is the area growing at time $t = 4$?

Example 8.9 (Student (if time)).

A right triangle is growing, with its vertical side growing at a constant rate of 1 unit per second, and its horizontal side growing at a constant rate of 2 units per second. At time $t = 2$ seconds, how fast is the hypotenuse growing?