We are concerned here with diffeomorphisms of a compact manifold $M$ whose nonwandering sets are hyperbolic.

Let $f$ be such a diffeomorphism and $\Omega$ its nonwandering set. Consider the following questions:

(a) Are the periodic orbits of $f$ dense in $\Omega$?
(b) Can $f$ be approximated by an $\Omega$-stable diffeomorphism?

We show in this chapter that both questions have a positive answer when $M$ is a closed two-dimensional manifold.

The first question was suggested by Smale in [7]. Related to it is Anosov's closing lemma: The periodic orbits are dense in the nonwandering set of $f/\Omega$, $f$ being as above. Also, one should notice that if $f$ is an Anosov diffeomorphism, then the periodic points are dense in the nonwandering set. This follows, for instance, from the structural stability of $f$ and Pugh's closing lemma.

With respect to the second question, if $f$ has the no-cycle property, then $f$ is $\Omega$-stable [8], [3]. On the other hand, if there is a cycle, one can radically change the nonwandering set by small perturbations, so $f$ is not $\Omega$-stable [5]. What we do here is to break all existing cycles without changing $\Omega$ to achieve an $\Omega$-stable diffeomorphism.

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We now set some notation, basic definitions, and the precise statements of our results.

Let $\text{Diff}^r(M)$ be the set of $C^r$ diffeomorphisms of $M$ with the $C^r$ topology, $r \geq 1$. For $f \in \text{Diff}^r(M)$, we denote by $P = P(f)$ the set of periodic points and by $\Omega = \Omega(f)$ the nonwandering set of $f$. We say that $\Omega = \Omega(f)$ is hyperbolic if there exist a continuous splitting $T_\omega M = E^s \oplus E^u$, a riemannian norm $| \cdot |$ on $TM$, and a constant $0 < \lambda < 1$ such that $|Tf(v)| \leq \lambda |v|$ for $v \in E^s$ and $|Tf^{-1}(v)| \leq \lambda |v|$ for $v \in E^u$. The diffeomorphism $g \in \text{Diff}^r(M)$ is called $\Omega$-stable if there exists a neighborhood $N$ of $g$ in $\text{Diff}^r(M)$ such that for any $g \in N$ there is a homeomorphism $h : \Omega(g) \rightarrow \Omega(g)$ satisfying the condition $hg(x) = gh(x)$ for all $x \in \Omega(g)$.

Assume $M$ is a compact manifold without boundary and $\dim M = 2$.

**Theorem A** For $f \in \text{Diff}^r(M)$, if $\Omega(f)$ is hyperbolic, then $P(f)$ is dense in $\Omega(f)$. That is, $f$ satisfies Smale's Axiom A.

**Theorem B** For $f \in \text{Diff}^r(M)$, if $\Omega(f)$ is hyperbolic, then $f$ can be approximated in $\text{Diff}^r(M)$ by an $\Omega$-stable diffeomorphism $g$ with $\Omega(g) = \Omega(f)$.

The starting point for proving these results is the analogue of the spectral decomposition theorem [7] for limit sets as considered in [3].

Let $f \in \text{Diff}^r(M)$. For $x \in M$, define

$$\omega(x) = \{y \in M : \text{there exists a sequence of integers } n_i \rightarrow \infty \text{ such that } \lim f^{n_i}(x) = y\}.$$ 

Dually, one defines $\alpha(x)$. Let

$$\omega(f) = \bigcup_{x \in M} \omega(x) \quad \text{and} \quad \alpha(f) = \bigcup_{x \in M} \alpha(x).$$

The limit set $L = L(f)$ of $f$ is the closure of $\omega \cup \alpha$.

If $L$ is hyperbolic, then $\bar{P} = L$, and $L$ can be written as the disjoint union $L = L_1 \cup \cdots \cup L_n$, each $L_i$ being closed, invariant by $f_i$ and topologically transitive. The periodic points are dense in each $L_i$, and $L_i$ has a local product structure, i.e., there is an $\varepsilon > 0$ such that $W^s_{\varepsilon}(L_i) \cap W^u_{\varepsilon}(L_i) \subset L_i([3], [7])$.

The $L_i$ are called basic limit sets.

Notice that

$$M = \bigcup_i W^s(L_i) = \bigcup_i W^u(L_i),$$
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and using [2]

\[ W^s(L_i) = \bigcup_{x \in L_i} W^s(x), \quad W^u(L_i) = \bigcup_{x \in L_i} W^u(x). \]

\(W^s\) and \(W^u\) stand, respectively, for the stable and unstable manifolds of sets involved. By \(W^s_{loc}(x)\) and \(W^u_{loc}(x)\), \(x \in L\), we mean \(W^s_\varepsilon(x)\) and \(W^u_\varepsilon(x)\), for some small \(\varepsilon > 0\) as defined in Section 2 of [2] (see also [1]).

If \(\dim W^s(x) = 1\), define \(W^s_+(x)\) and \(W^s_-(x)\) to be the components of \(W^s_{loc}(x) - \{x\}\).

From now on, we assume \(\dim M = 2\) and \(f \in \text{Diff}^r(M)\) with \(\Omega = \Omega(f)\) hyperbolic.

**Proposition 1** There exists a finite subset \(F^s \subset P\) such that if \(x \in L\), \(\dim W^s(x) = 1\) and \(x\) is not a limit point of both \(W^s_+(x) \cap L\) and \(W^s_-(x) \cap L\), then \(x \in W^u(p), p \in F^s\).

**Proof** Let \(\varepsilon > 0\) be such that no arc \(V \subset W^s_{loc}(x)\) with both boundary points in \(L\) and length less than \(\varepsilon\) contains \(x\) in its interior.

We first show that \(x \in W^u(p)\), for some \(p \in P\). Suppose not. Since \(x \in L\), \(x \in L_i\) for some basic limit set \(L_i\). Let \(y \in \alpha(x)\) and \(n_i\) a sequence of integers such that \(n_i \to \infty\) and \(f^{-n_i}(x) \to y\). Clearly \(y \in L_i\) and using the local product structure of \(L_i\) we get an integer \(n > 0\) and an arc \((z, w) \subset f^{-n}W^s_{loc}(x)\) of length smaller than \(\varepsilon\) containing \(f^{-n}(x)\) in its interior, \(z\) and \(w\) in \(L_i\). Thus we reach a contradiction, since \(f^n(z), f^n(w) \in L_i\), and the arc \((f^n(z), f^n(w)) \subset W^s_{loc}(x)\) has length less than \(\varepsilon\) and contains \(x\) in its interior.

A similar argument shows that the subset of \(P\) for which there are points in their unstable manifolds satisfying the hypotheses of the proposition is finite.

The same fact is true interchanging stable and unstable manifold in the above proof. This yields a corresponding finite subset \(F^u \subset P\). We denote by \(F\) the union of \(F^s\) and \(F^u\).

We now define basic regions and a partial ordering relation on them. This will play a key role in proving the main results in the paper.

Let \(p \in F \cap L_i\) with period \(n\), \(L_i\) being a basic limit set. Clearly, \(p\) is a saddle point. Let \(U, f(U), \ldots, f^{n-1}(U)\) be a pairwise disjoint small open disks containing the orbit \(O(p)\) such that in \(U' = \bigcup_{k=0}^{n-1} f^k(U)\), \(W^s_{loc}(O(p))\) and \(W^u_{loc}(O(p))\) are well defined and \(U' \cap L_j = \emptyset\) for \(j \neq i\).

In \(U'\) we distinguish the open regions \(R_1, \ldots, R_m\) bounded by \(W^s_{loc}(O(p))\) and \(W^u_{loc}(O(p))\) such that \(U' \cap f^l(R_k) \subset R_k\) for \(0 \leq l \leq n\). The \(R_k\)
are called basic regions. For instance, if \((T f^n)_p\) has positive eigenvalues then there are four basic regions \(R_1, R_2, R_3, R_4\) associated to \(O(p)\), each one has \(n\) connected components corresponding to the open quadrants defined in \(U'\) by \(W^s_{\text{loc}}(O(p))\) and \(W^u_{\text{loc}}(O(p))\). If one eigenvalue of \((T f^n)_p\) is negative, then there are two basic regions, each having \(2n\) connected components.

Notice that \(W^s_{\text{loc}}(O(p)) \cap R_k = \emptyset\) and \(W^u_{\text{loc}}(O(p)) \cap R_k = \emptyset\) for each \(k\).

For a basic limit set \(L_j\), which is not a periodic orbit in \(F\), the basic region is defined simply as a neighborhood \(R\) of \(L_j\), where \(W^s_{\text{loc}}(L_j)\) and \(W^u_{\text{loc}}(L_j)\) are well defined and \(R \cap L_r = \emptyset\) for \(r \neq j\).

Let \(R_1, R_2, \ldots, R_n\) be the basic regions considered above. In \(R_i\) we define

\[
R^s_i = W^s_{\text{loc}}(O(p)) \cap \bar{R}_i - L, \\
R^u_i = W^u_{\text{loc}}(O(p)) \cap \bar{R}_i - L,
\]

or

\[
R^s_i = W^s_{\text{loc}}(L_i) - \{W^n(F \cap L_i) \cup L_i\}, \\
R^u_i = W^u_{\text{loc}}(L_i) - \{W^n(F \cap L_i) \cup L_i\},
\]

according to whether \(R_i\) is associated to \(O(p) \subset F\) or to a basic limit set \(L_i\) not contained in \(F\).

Notice that the correspondences \(R_i \rightarrow R^s_i\) and \(R_i \rightarrow R^u_i\) may be two-to-one for periodic orbits in \(F\).

We also globalize these "local components" of stable and unstable manifolds by setting

\[
\bar{R}^s_i = \bigcup_{0}^{\infty} f^{-n}(R^s_i), \\
\bar{R}^u_i = \bigcup_{0}^{\infty} f^{n}(R^u_i).
\]

Given two basic regions \(R_i\) and \(R_j\), we say that \(R_i\) almost visits \(R_j\) if there exist sequences \(x_n \to x, x_n \in R_i\) and \(x \in R^s_j\), and \(m_n \to \infty\), such that \(f^{m_n}(x_n) \to y, f^{m_n}(x_n) \in R_j\), and \(y \in R^s_j\). In this case, we also say that \(x\) almost visits \(y\) in \(R_j\) from \(R_i\).

**Lemma 2** Let \(R_i\) and \(R_j\) be basic regions such that \(R_i\) almost visits \(R_j\). Then, given any open set \(V\) meeting \(R^s_i\), we have \(R^u_j \subset \bigcup_{n>0} f^n(V)\).

**Proof** We construct a sequence of basic regions \(R_{i_0} = R_i, R_{i_1}, \ldots, R_{i_i} = R_j\), and a sequence of points \(x_0, x_1, \ldots, x_i\) such that
(a) $R_{ik}$ almost visits $R_{ik+1}$ for $0 \leq k < l$;

(b) Either $x_{k+1} \in R_{ik} \cap R_{ik+1}^u$ or $x_{k+1} \in W^u(q) \cap R_{ik+1}^s$, where $q \in F \cap L_k$ and $R_{ik}$ is associated to some $O(p) \subset F \cap L_k$ or to $L_k$, $0 \leq k < l$;

(c) For any open set $V_k$ meeting $R_{ik}^s$, $R_{ip}^u \subset \bigcup_{n \geq 0} f^n(V_k)$ for $0 \leq k \leq p \leq l$.

Such a sequence, whose existence will be proved by induction, implies the lemma.

Since $R_i$ almost visits $R_j$, we have sequences $x_n \to x_0$, $x_n \in R_i$ and $x_0 \in R_j^s$, and $f^{m_n}(x_n) \to y$, with $m_n \to \infty$, $f^{m_n}(x_n) \in R_j$, and $y \in R_j^s$. Take $R_{i_0} = R_i$. Let $U$ be a small neighborhood of $L_i$, $R_i$ being associated to $L_i$ or to $O(p) \subset F \cap L_i$. From [2], there exist $x_i \in W^u_{loc}(L_i) - L_i$ and a sequence $\varepsilon_n < m_n$ such that $\varepsilon_n \to 0$, $f^i(x_i) \in U$, for $0 \leq j \leq \varepsilon_n$, and $f^{m_n}(x_n) \to x_i$. Either $x_i \in W^u(L_i) - W^u(F \cap L_i)$ or $x_i \in W^u(q)$ for some $q \in F \cap L_i$. Now we can find an index $i$, such that $x_i \in R_i^s$, $R_i$ almost visits $R_{i_1}$ and $R_{i_1}$ almost visits $R_j$, and given any neighborhood $V$ meeting $R_{i_1}^s$, $R_{i_1}^u \subset \bigcup_{n \geq 0} f^n(V)$. In the case $x_1 \in W^u(L_i) - W^u(F \cap L_i)$, this is easily checked since $W^u(L_i) \subset \bigcup_{n \geq 0} f^n(V)$ and $R_{i_1}^u \subset \overline{W^u(L_i)}$ for the possible indices $i_1$ such that $x_1 \in R_i^s$. If $x_1 \in W^u(F \cap L_i)$, then $W^u(x_1) \subset \overline{W^u_{loc}(L_i)}$. Of course, $W^u(x_1)$ and $W^s(x_1)$ might meet transversely at $x_1$ or not. Analyzing the types of possible intersections of $W^u(x_1)$ and $W^s(x_1)$ at $x_1$ and using the $\lambda$-lemma [4], one can choose $R_{i_1}$ with $x_1 \in R_{i_1}^s$ as claimed. Once $x_k$, $R_{ik}$ have been chosen, a similar construction yields a point $x_{k+1}$ and a region $R_{ik+1}$ with $x_{k+1} \in R_{ik+1}^s$, so that conditions (a)–(c) above hold for the sequence $R_{i_1}, \ldots, R_{ik+1}$.

Finally, we claim that we must reach $R_j$ with this process. In fact, the set of basic regions is finite, so the only possible obstruction to that would be the existence of cycles in the sequence, i.e., sequences $R_{im}$, $R_{im+1}$, $\ldots$, $R_{in}$ as above with $R_{im} = R_{in}$. But if this happens, condition (c) implies that all the points $x_k \in R_{ik}^s$, $m \leq k < n$, are nonwandering. Since $\Omega$ is hyperbolic, $W^u(x_k)$ and $W^s(x_k)$ meet transversely at $x_k$. This implies that each $x_k$ is in the closure of the set of transversal homoclinic points of the periodic points. It now follows from [6] that $x_k \in \overline{P} = L$, contradicting the construction of $x_k$. Thus the lemma is proved.

For basic regions $R_i$ and $R_j$, define $R_i \geq R_j$ if either $R_i = R_j$ or $R_i$ almost visits $R_j$.

As an immediate consequence of Lemma 2 we have

**Theorem 3** The relation $\geq$ defines a partial ordering on the set of basic regions.
We can now prove Theorem A.

**Proof of Theorem A** Since $\bar{P} = L$, it is enough to prove $\Omega = L$. Suppose there exists $x \in \Omega - L$. Then $x \in \bar{R}_i \cap \bar{R}_j^u$ for basic regions $R_i, R_j$, and $R_i$ almost visits $R_j$. We now proceed as in Lemma 2 constructing a sequence of basic regions $R_{i_0} = R_i, R_{i_1}, \ldots, R_{i_l} = R_j$, and a sequence of points $x_0 = x, x_1, \ldots, x_l$ such that

(a) $R_{i_k}$ almost visits $R_{i_{k+1}}$, for $0 \leq k < l$;
(b) Either $x_{k+1} \in R_{i_k} \cap \bar{R}_{i_{k+1}}$ or $x_{k+1} \in W^u(q) \cap \bar{R}_{i_k}$, where $q \in F \cap L_k$ and $R_{i_k}$ is associated to some $O(p) \subset F \cap L_k$ or to $L_k$, $0 \leq k < l$;
(c) For any open set $V_k$ meeting $R_{i_k}, R_{i_k}^u \subset \bigcup_{n=0}^{\infty} f^n(V_k)$ for $0 \leq k \leq p \leq l$.

Also, since $x \in \Omega$ and $\Omega$ is hyperbolic,
(d) $W^u(x)$ and $W^s(x)$ meet transversely at $x$.

Thus taking any neighborhood $V$ of $x_1$, $\bar{R}_{i_1} = \bigcup_{n=0}^{\infty} f^n(V)$, and using (d), $\bigcup_{n=0}^{\infty} f^n(V) \cap V \neq \emptyset$, which implies that $x_1 \in \Omega$. Again since $\Omega$ is hyperbolic, $W^u(x_1)$ and $W^s(x_1)$ meet transversely at $x_1$. Repeating the argument, it follows that each $x_k$ is nonwandering and consequently $W^u(x_k)$ and $W^s(x_k)$ meet transversely at $x_k, 0 \leq k < l$. Again, from [6], $x \in \bar{P} = L$, which contradicts the assumption that $x \in \Omega - L$. Therefore, $\Omega = L$ and so Theorem A is proved.

As a consequence of Theorem A, we have that each basic limit set $L_i$ in the spectral decomposition theorem corresponds, in fact, to a basic set $\Omega_i$ as defined by Smale in [7]. We keep the notation $\Omega = L = L_1 \cup \cdots \cup L_n$.

As in [3], define the relation $\geq$ on the basic sets by $L_i \geq L_j$ if there is a sequence $L_{i_0} = L_i, L_{i_1}, \ldots, L_{i_l} = L_j$ such that $W^u(L_{i_k}) \cap W^s(L_{i_{k+1}}) \neq \emptyset$ for $0 \leq k < l$. A corresponding equivalence relation is defined on the $\{L_i\}$ by $L_i \sim L_j$ if $L_i \geq L_j$ and $L_j \geq L_i$. Let $\{\gamma_1, \ldots, \gamma_m\}$ be the distinct equivalence classes of $\{L_i\}$ under $\sim$. The $\gamma_i$ are partially ordered by $\gamma_i \geq \gamma_j$ if there exist $L_i \in \gamma_i$ and $L_j \in \gamma_j$ such that $L_i \geq L_j$.

In what follows, the union of the basic sets in a class $\gamma_i$ will also be denoted by $\gamma_i$.

We now relate these notions with basic regions and stable and unstable manifolds of basic regions.

A component of the class $\gamma$ is a global unstable manifold $\bar{R}_k^u$ of the basic region $R_k$, where $R_k$ is associated to a periodic orbit in $F \cap \gamma$. The component $\bar{R}_k^u$ is free if $\bar{R}_k^u \subset \bigcup_{\gamma_i \neq \gamma_j} W^s(\gamma_j)$. 

Lemma 4 Suppose that \( \tilde{R}_k^u \subset W_u(\gamma) \) and \( \tilde{R}_k^u \cap W^s(\gamma) \neq \emptyset \) for some equivalence class \( \gamma \). Then there are basic regions associated to \( \gamma \), \( R_{k_0} = R_k > R_{k_1} > \cdots > R_{k_n} \) and \( R_{k_{n+1}}, R_{k_{n+2}}, \ldots, R_{k_{n+m}} \) such that

(a) \( R_{k_n} > R_{k_n+r} \) for \( 1 \leq r \leq m \);
(b) \( \tilde{R}_{k_{n+r}} \) is free for \( 1 \leq r \leq m \);
(c) \( \tilde{R}_{k_n}^u \subset \bigcup_{r=1}^m W^u(O(p_r)) \cup \bigcup_{\gamma_j < \gamma} W^s(\gamma_j) \), and
\[
(\tilde{R}_{k_n}^u - (\tilde{R}_{k_n} \cup L_{k_n})) \cap W^u(\gamma) \subset \bigcup_{r=1}^m \tilde{R}_{k_{n+r}}^u \cup O(p_r),
\]
where \( L_{k_n} \) is the basic set to which \( R_{k_n} \) is associated, and \( O(p_r) \) is the periodic orbit of \( F \cap \gamma \) to which \( R_{k_{n+r}} \) is associated.
(d) \( \tilde{R}_{k_{n+r}} \) is connected, for \( 1 \leq r \leq m \).

Proof The lemma is proved by induction on the number of basic regions involved. Suppose \( R_{k_0} = R_k > R_{k_1} > \cdots > R_{k_j} \) has been defined. If there are no \( R_{k_{j+1}}, \ldots, R_{k_{j+a}} \) such that conditions (a)–(d) are satisfied, we can proceed to construct \( R_{k_{j+1}} \) such that
\[
R_{k_0} = R_k > R_{k_1} > \cdots > R_{k_j} > R_{k_{j+1}}.
\]
Since the number of basic regions is finite and by Theorem 3 they are partially ordered by \( > \), eventually we get a sequence of basic regions as desired.

Corollary 5 In the previous lemma the basic region \( R_{k_n} \) is associated to a periodic orbit in \( F \cap \gamma \).

From Lemma 4 and Corollary 5, it follows that if each class \( \gamma_i \) has only free components, then \( f \) has the no-cycle property and thus is \( \Omega \)-stable ([3, 8]). Theorem B will be proved through a finite number of small perturbations that keep the nonwandering set the same and increase the number of free components.

We now restrict our attention to the periodic orbits \( O(p_1), \ldots, O(p_r) \) that occur in Lemma 4.

Let \( \gamma \) be a \( \sim \)-class consisting of more than one basic set. Let \( R_{i_1} \) and \( R_{i_2} \) be basic regions associated to a periodic orbit \( O(p) \) in \( F \cap L_i \), \( L_i \in \gamma \), so that \( \tilde{R}_{i_1}^u = \tilde{R}_{i_2}^u \) is free. Suppose \( R_{i_3} \) and \( R_{i_4} \) are the other basic regions associated to \( O(p) \), \( R_{i_3} \) adjacent to \( R_{i_1} \) and \( R_{i_4} \) adjacent to \( R_{i_2} \), i.e., \( \tilde{R}_{i_3} = \tilde{R}_{i_1}^u \) and \( \tilde{R}_{i_4} = \tilde{R}_{i_2}^u \). Notice that possibly \( i_1 = i_2 \) and
$i_3 = i_4$. For $\varepsilon > 0$ small and $\delta < \varepsilon/2$, let $G = G' \cap W^s_\varepsilon(O(p))$, where

$$G' = \overline{W^s_\delta(L_4)} - f\overline{W^s_\delta(L_3)},$$

i.e., $G'$ is a proper fundamental domain for $L_i$ [2].

**Lemma 6** There exists a neighborhood $N$ of $G$ such that for any $y \in M - \Omega$ and $x', x'' \in N \cap \{\mathcal{R}_i \cup \mathcal{R}_3\}$ if $x'$ almost visits $y$ from $R_{i_3}$, then $y$ can only visit $x''$ in $R_{i_4}$.

The same is true interchanging $R_{i_4}$ and $R_{i_3}$.

**Proof** If the lemma were false, choosing appropriate convergent subsequences, we could find $x', x'' \in R^k_{i_3} \cap N$ and $y \in M - \Omega$ such that $x'$ almost visits $y$ from $R_{i_3}$ and $y$ almost visits $x''$ in $R_{i_4}$. Since $y \notin \Omega = L$, we can choose $R_{j_1}$ and $R_{j_2}$ such that $\mathcal{R}_{j_1} \cap \mathcal{R}_{j_2} \ni y$, $R_{j_1}$ almost visits $R_{i_3}$ and $R_{i_4}$ almost visits $R_{j_2}$. Therefore, $R_{j_1}$ almost visits $R_{j_2}$ by Theorem 3. Moreover, constructing a sequence as in Lemma 2, we see that $V \cap \bigcup_{n>0} f^n(V) \neq \emptyset$ for any neighborhood $V$ of $y$. Thus $y \in \Omega$, contradicting the assumption. Lemma 6 is proved.

Notice that if we perform a perturbation of $f$ with support in $N$ that creates a new nonwandering point $y \in M$, then there must exist a pair of points $x', x''$ as above.

**Proof of Theorem B** The theorem is proved by making a sequence of approximations, each increasing the number of free components, without changing $\Omega$.

We can take on $\{\gamma_i\}$ a linear ordering compatible with $\succeq$, so that $\gamma_n \succeq \gamma_{n-1} \succeq \cdots \succeq \gamma_1$. Suppose all components of $\gamma_j$ are free for $\gamma_j \preceq \gamma_{i-1}$, so each class $\gamma_j$ is made of a unique basic set. Suppose $\mathcal{R}_{k_0}^u$
is associated to the class $y_i$ and $\mathcal{R}_k^u \cap W^s(y_i) \neq \emptyset$. From Lemma 4, there are regions $R_{k_n}, R_{k_{n+1}}, \ldots, R_{k_{n+m}}$ such that $R_k > R_{k_i}$ for $n \leq i \leq m + n$, and (a)-(d) in the lemma are satisfied.

Using Lemma 6, we can perform an arbitrarily small $C^r$ perturbation of $f$ such that $\Omega$ remains the same, the free components of $\gamma_j$ remain free for $j \leq i$, and $\mathcal{R}_k^u$ becomes free. This perturbation has support in the fundamental neighborhoods $N_r, 1 \leq r \leq m,$ defined in Lemma 6 and is taken so small as to be disjoint from all free components of $\gamma_j,$ for $j \leq i.$ For each $r, 1 \leq r \leq m,$ the perturbation is the time-one map of a vector field parallel to $\mathcal{R}_{k_{n+r}}^u$ with the same "orientation," as described in Figure 1.

In this way, we achieve a diffeomorphism $g, C^r$ near $f,$ such that $\Omega(g) = \Omega(f)$ and all the components of the basic sets for $g$ are free. Therefore $g$ has the no-cycle property and by [8] is $\Omega$-stable.

References


