Exam Tips:

- The exam will cover sections 16.4 (spherical coordinates), 16.5, 16.6, 17.1, 17.2, and 17.3.
- Know the hypothesis and conclusions for all theorems covered so far.
- Know all relevant definitions.
- Review one-variable integration techniques (such as u substitution and integration by parts) as well as the anti-derivatives of common functions.
- In addition to the problems provided here, it would be a good idea to do some additional exercises from the textbook (use the homework as an indication of which problem types are relevant),

Practice Problems:

1. Let $\mathcal{W}$ be the region in $\mathbb{R}^3$ determined by

$$\mathcal{W}\colon \begin{cases} z^2 - x^2 - y^2 \geq 0 \\ x^2 + y^2 + z^2 \leq 1 \\ y \geq 0 \end{cases} .$$

   (a) Describe $\mathcal{W}$ using spherical coordinates (ρ, ϕ, θ).

   (b) Compute $\iiint_{\mathcal{W}} \sqrt{x^2 + y^2 + z^2}\, dV$.

2. Let $\mathcal{W} = [0,1] \times [0,1] \times [0,1]$ and suppose $\rho(x, y, z) = e^{x+y+z}$ is a density function for $\mathcal{W}$.

   (a) Find the total mass of the cube $\mathcal{W}$.

   (b) Find the coordinates $(x_{\text{CM}}, y_{\text{CM}}, z_{\text{CM}})$ for the center of mass of $\mathcal{W}$.

3. Let $\mathcal{D}$ be the region bounded by the curves

$$\mathcal{D}\colon \begin{cases} y = 2x + 1 & y = 2x - 2 \\ y = -x + 1 & y = -x + 3 \end{cases} .$$

   Let G be the following change of variables map

$$G(u, v) = \left(\frac{u - v}{3}, \ \frac{2u + v}{3} \right),$$

   and let $\mathcal{D}_0$ be the region in the uv-plane that $G(u, v)$ maps onto $\mathcal{D}$.

(a) Sketch the regions $\mathcal{D}_0$ and $\mathcal{D}$. Be sure to label your axes, the boundary curves, and any points where the boundary curves intersect.

(b) Compute the double integral $\iint_{\mathcal{D}} x + y \, dA$.

4. Let $\mathcal{C}$ be the oriented curve with the following parametrization

$$\vec{c}(t) = \left\langle t,\ t^2\, t^3 \right\rangle \qquad 0 \leq t \leq 2.$$

Compute $\int_{\mathcal{C}} \vec{F} \cdot d\vec{s}$ for the vector field

$$\vec{F}(x, y, z) = \langle e^x, e^y, e^z \rangle .$$

5. Consider the oriented curve $\mathcal{C}$ with parametrization

$$\vec{c}(t) = \left\langle \frac{\sqrt{2}}{2} \sin t,\ -\frac{\sqrt{2}}{2} \sin t,\ \cos t \right\rangle \qquad 0 \leq t \leq 2\pi.$$

(a) Show that the $\mathcal{C}$ is the intersection of the plane $x + y = 0$ and the the sphere $x^2 + y^2 + z^2 = 1$ by showing that $\vec{c}(t)$ gives points which are always on both the plane and the sphere.

(b) Let $\vec{F}(x, y, z) = \langle 1, 1, 0 \rangle$ be a constant vector field. Compute $\int_{\mathcal{C}} \vec{F} \cdot d\vec{s}$.

(c) Let

$$\vec{F}(x, y, z) = \left\langle \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \frac{z}{\sqrt{x^2 + y^2 + z^2}} \right\rangle .$$

Compute $\int_{\mathcal{C}} \vec{F} \cdot d\vec{s}$.

Hint: for parts (b) and (c) you do not actually need to compute anything.

6. Let $\mathcal{C}_1$ be the line segment starting at $(-1, 0)$ and going to $(1, 0)$, and let $\mathcal{C}_2$ be the top half of the circle $x +^2 +y^2 = 1$, oriented counter-clockwise. Consider the vector field

$$\vec{F}(x, y) = \left\langle \frac{1}{1 + y^2},\ \frac{-2xy}{(1 + y^2)^2} \right\rangle .$$

(a) Compute $\int_{\mathcal{C}_1} \vec{F} \cdot d\vec{s}$.

(b) Compute $\int_{\mathcal{C}_1 \cup \mathcal{C}_2} \vec{F} \cdot d\vec{s}$.