Exam Tips:

- The exam will cover sections 16.1, 16.2, 16.3, and 16.4 (excluding spherical coordinates).
- Know the hypothesis and conlcusions for all theorems covered so far.
- Know all relevant defintions.
- Review one-variable integration techniques (such as u substitution and integration by parts) as well as the anti-derivatives of common functions.
- In addition to the problems provided here, it would be a good idea to do some additional exercises from the textbook (use the homework as an indication of which problem types are relevant),

Practice Problems:

1. Let $\mathcal{D}$ be the region bounded by the curves

$$\begin{cases} y = (1-e)x + 1 & \text{and} \\ y = e^{-x} \end{cases}.$$

Note that these curves intersect at the points $(0, 1)$ and $(-1, e)$ and that $1 - e < 0$.

   (a) Sketch the region $\mathcal{D}$. Be sure to label your axes, the boundary curves, and any points where the boundary curves intersect.

   (b) Compute $\displaystyle\iint_{\mathcal{D}} y^2 \exp\left(x + \frac{y}{e-1}\right)\, dA$.

2. Suppose the double integral $\displaystyle\iint_{\mathcal{D}} ye^x\, dA$ over a region $\mathcal{D}$ is equivalent to the following iterated integral:

$$\int_0^6 \int_0^{g(x)} ye^x\, dy\, dx,$$

where

$$g(x) = \begin{cases} \sqrt{x} & \text{if } 0 \le x \le 4 \\ 6 - x & \text{if } 4 \le x \le 6 \end{cases}.$$

   (a) Sketch the region $\mathcal{D}$. Be sure to label your axes, the boundary curves, and any points where the boundary curves intersect.

   (b) Write a presentation of $\mathcal{D}$ as a horizontally simple region.

(c) Compute $\displaystyle\iint_{\mathcal{D}} ye^x \, dA$.

3. Let $\mathcal{W}$ be the region in the first octant $(x, y, z \geq 0)$ bounded by the planes

$$\begin{aligned} 4x - 2y + 2z &= 10 \qquad \text{and} \\ -2x - 5y + z &= 1. \end{aligned}$$

Find the volume of $\mathcal{W}$. (*Hint: integrate with respect to z first.*)

4. The polar region $\mathcal{D}$ bounded by the curve $r = \cos(5\theta)$, $0 \leq \theta \leq \pi$ is a 5-petal flower.

(a) Use the change of variables formula for polar coordinates to find the area of a single petal.

(b) Compute $\iint_{\mathcal{D}} \sin(5\theta) \, dA$.

5. The region $\mathcal{W}$ bounded by the surfaces

$$\begin{cases} z = \frac{b}{a} r & \quad (0 \leq \theta \leq 2\pi) \\ z = b \end{cases}$$

is a right cone of radius a and height b (vertex pointing downward at the origin). Use the change of variables formula for integrating in cylindrical coordinates to produce the well known formula for the volume of such a cone: $\frac{1}{3}\pi a^2 b$.