Math 105 | **Syllabus** | **Spring 2018**

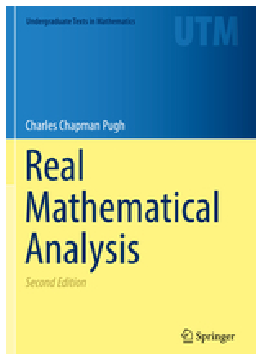
Instructor
Brent Nelson
Evans 851
brent [at] math.berkeley.edu

Lecture
MWF
12:00 - 1:00 pm
3 Evans

Office Hours: Tuesdays 2:30 - 4:30 pm, Fridays 2:30 - 3:30 pm, and by appointment.

Course Webpage: `http://www.math.berkeley.edu/~brent/105.html`

Textbook: Charles C. Pugh, *Real Mathematical Analysis*, Springer UTM



Using your UC Berkeley credentials, you should be able to download a free copy of this book here. We will cover Chapters 5 and 6.

Course Description: In this course we will produce a rigorous foundation for multi-variable calculus. The official course description can be found here.

In Math 104, you learned that if a function $f\colon (a,b) \to \mathbb{R}$ is differentiable at a point $x_0 \in (a,b)$, then the derivative helps us find a suitable *linear* approximation for f; that is,

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0),$$

for x close to x_0. In Math 105, we will consider the more general class of functions $f\colon \mathbb{R}^n \to \mathbb{R}^m$, $n, m \in \mathbb{N}$, and our first goal is determine what it means for such a function to be differentiable. It turns out that the previous way of thinking generalizes to this new situation: the derivative should be something that helps us find a linear approximation for f. Thanks to linear algebra, we know quite a lot already about linear functions from $\mathbb{R}^n$ to $\mathbb{R}^m$; namely, we can always represent such functions as $m \times n$ matrices. Thus the course will begin with a (brief) review of linear algebra.

Once we have made this definition of the derivative rigorous, we will explore its implications and see how to generalize some of the single-variable theorems you learned in Math 104 (e.g. the chain rule, the Leibniz rule, the mean value theorem, etc.). We will also make sense of higher order derivatives in this context. We will conclude our examination of the derivative with the (very standard) implicit and inverse functions theorems.

We will define the Riemann integral for functions of the form $f\colon \mathbb{R}^m \to \mathbb{R}$ using an almost identical definition to that seen in Math 104. After ironing out some technical details, we determine and prove the change of variables formula in this context. Our ultimate goal in this section (and the main result in Chapter 5) is the general form of Stokes' theorem.

Towards this end we will engage in a lengthy study of *differential forms*, which will give a rigorous meaning to the notation 'dx.'

Changing gears slightly in Chapter 6, we will consider an alternative to the Riemann integral: the *Lebesgue integral*. The definition of the Riemann integral in terms of Riemann sums emphasizes the importance of step functions and intervals, whereas the Lebesgue integral is defined in such a way to emphasize *simple functions* and *measurable sets*. This broader class of functions and sets is what makes the Lebesgue integral more robust: any Riemann integrable function is Lebesgue integrable, whereas functions can be Lebesgue integrable but **not** Riemann integrable. Understanding these objects will require an excursion into measure theory, though this will not be as general as one might see in Math 202A. Time permitting, we will cover all of Chapter 6 and possibly some L^2 theory. However, unlike in Chapter 5 where we will faithfully follow the text, in Chapter 6 we may diverge from the text in terms of both material and notation.

In-Class Tone: My aim is to foster an open and inclusive atmosphere in class. Therefore questions, participation, collaboration, and curiosity are strongly encouraged. Math can be hard, especially when we aren't honest with ourselves about whether or not we understand something. Confusion is not a sign of weakness, nor is asking for help. If you need help beyond class time and office hours, please do not hesitate to contact me so that we can work out additional times to meet.

Homework: There will be a total of 12 homework assignments. These will be posted on the course webpage, and will be collected at the beginning of lecture on Wednesdays. No late homework will be accepted. The lowest two homework scores will be automatically dropped. Collaboration is allowed (encouraged even), but your written proofs must clearly be your own and indicate that you understand the argument

Midterms: The course will have two midterm examinations:

Midterm 1	Wednesday, February 14th
Midterm 2	Friday, March 23rd

No make-up exams will be offered (see grading policy below). Please check early in the semester to make sure you have no time conflicts with these exams.

Final: The final exam is on Wednesday, May 9th from 3:00 - 6:00 pm. You must take the final exam to pass the class. Please bring your Cal 1 Card with you to the final exam.

Grading: There will be two grading schemes offered and I will automatically select the one which gives you the best grade. They are as follows:

	Homeworks	**Midterms**	**Final Exam**
Scheme 1:	20% (2% each)	40% (20% each)	40%
Scheme 2:	20% (2% each)	20% (best one)	60%

Letter grades will only be assigned at the end of the course once a suitable distribution can be determined. Your overall raw score will always earn you a grade **at least as good as** the traditional grade buckets (e.g. 90% and above will earn at least an 'A-', 80% and above will earn at least a 'B-,' etc.).

The following bCourses site will be used to maintain a gradebook for the course:

https://bcourses.berkeley.edu/courses/1468516/gradebook

If you believe there is an error with the grading of any course material, you must notify the instructor within 14 calendar days of when it was completed, otherwise it will not be given further consideration. The general course policies for the University of California, Berkeley can be found here.

Course Calender: The following is a schedule for the course (subject to change).

Week 1	1/15 - 1/19	No class on 1/15 Homework 1 due on 1/19
Week 2	1/22 - 1/26	Homework 2 due on 1/24
Week 3	1/29 - 2/2	Homework 3 due on 1/31
Week 4	2/5 - 2/9	Homework 4 due on 2/7
Week 5	2/12 - 2/16	**Midterm 1** on 2/14
Week 6	2/19 - 2/23	No class on 2/19 Homework 5 due on 2/21
Week 7	2/26 - 3/2	Homework 6 due on 2/28
Week 8	3/5 - 3/9	Homework 7 due on 3/7
Week 9	3/12 - 3/16	Homework 8 due on 3/14
Week 10	3/19 - 3/23	**Midterm 2** on 3/23
Week 11	3/26 - 3/30	No classes (Spring Break)
Week 12	4/2 - 4/6	Homework 9 due on 4/4
Week 13	4/9 - 4/13	Homework 10 due on 4/11
Week 14	4/16 - 4/20	Homework 11 due on 4/18
Week 15	4/23 - 4/27	Homework 12 due on 4/25
Week 16	4/30 - 5/4	No class (Reading/Review/Recitation Week)
Finals Week	5/7 - 5/11	**Final Exam** on 5/9 (3:00 - 6:00 pm)