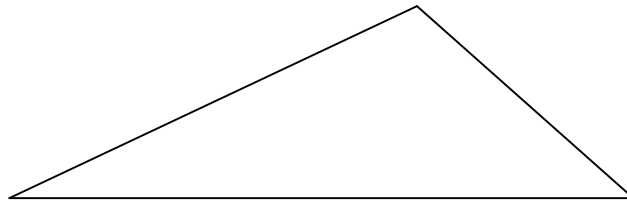


The 3rd MidMichigan Mathematical Olympiad
April 21, 2005

Grades 5-6

1. Is there an integer such that the product of all whose digits equals 99?
2. An elevator in a 100 store building has only two buttons: UP and DOWN. The UP button makes the elevator go 13 floors up, and the DOWN button makes it go 8 floors down. Is it possible to go from the 13th floor to the 8th floor?
3. Cut the triangle shown in the picture into three pieces and rearrange them into a rectangle. (Pieces can not overlap.)

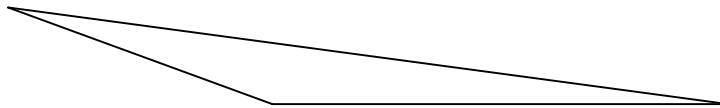


4. Two players Tom and Sid play the following game. There are two piles of rocks, 5 rocks in the first pile and 6 rocks in the second pile. Each of the players in his turn can take either any amount of rocks from one pile or the same amount of rocks from both piles. The winner is the player who takes the last rock. Who does win in this game if Tom starts the game?
5. In the next long multiplication example each letter encodes its own digit. Find these digits.

$$\begin{array}{r} \text{a b} \\ * \text{c d} \\ \hline \text{c e f} \\ + \text{a b} \cdot \\ \hline \text{c f d f} \end{array}$$

The 3rd MidMichigan Mathematical Olympiad
April 21, 2005
Grades 7-8

1. Prove that no matter what digits are placed in the four empty boxes, the eight-digit number $9999\square\square\square\square$ is not a perfect square. (A perfect square is a number which equals to an integer times itself. For example, 25 is a perfect square because $25=5 \times 5$.)
2. Prove that the number $m/3+m^2/2+m^3/6$ is integral for all integral values of m .
3. An elevator in a 100 store building has only two buttons: UP and DOWN. The UP button makes the elevator go 13 floors up, and the DOWN button makes it go 8 floors down. Is it possible to go from the 13th floor to the 8th floor?
4. Cut the triangle shown in the picture into three pieces and rearrange them into a rectangle. (Pieces can not overlap.)



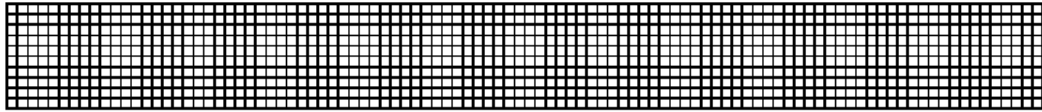
5. Two players Tom and Sid play the following game. There are two piles of rocks, 7 rocks in the first pile and 9 rocks in the second pile. Each of the players in his turn can take either any amount of rocks from one pile or the same amount of rocks from both piles. The winner is the player who takes the last rock. Who does win in this game if Tom starts the game?

6. In the next long multiplication example each letter encodes its own digit. Find these digits.
- $$\begin{array}{r}
 \text{a b} \\
 * \text{ c d} \\
 \hline
 \text{c e f} \\
 + \text{ a b} \\
 \hline
 \text{c f d f}
 \end{array}$$

The 3rd MidMichigan Mathematical Olympiad
April 21, 2005

Grades 10-12

1. A tennis net is made of strings tied up together which make a grid consisting of small squares as shown below.



The size of the net is 100x10 small squares. What is the maximal number of sides of small squares which can be cut without breaking the net into two separate pieces? (The side is cut only in the middle, not at the ends).

2. What number is bigger 2^{300} or 3^{200} ?
3. All noble knights participating in a medieval tournament in Camelot used nicknames. In the tournament each knight had combats with all other knights. In each combat one knight won and the second one lost. At the end of tournament the losers reported their real names to the winners and to the winners of their winners. Was there a person who knew the real names of all knights?
4. Two players Tom and Sid play the following game. There are two piles of rocks, 10 rocks in the first pile and 12 rocks in the second pile. Each of the players in his turn can take either any amount of rocks from one pile or the same amount of rocks from both piles. The winner is the player who takes the last rock. Who does win in this game if Tom starts the game?
5. There is an interesting 5-digit integer. With a 1 after it, it is three times as large as with a 1 before it. What is the number?