Math 21C-2
Practice Midterm

Name:______________________________________

Signature:__________________________________

Student ID:_________________________________

- There are **ten** (plus cover and bonus) pages to the exam.
- The exam totals 100 points, plus 10 bonus points.
- You will have 90 minutes to complete the exam.
- No calculators, notes, or books allowed.
- Good luck!

Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Bonus	10	
Total	100	

1. (10 points) Definitions and Examples:

a. (2 points) Write the definition for a sequence to be *bounded above*. (An example is **not** sufficient for full credit.)

b. (2 points) Write the definition of an *alternating series*. (An example is **not** sufficient for full credit.)

c. (2 points) Write the definition for a sequence to *diverge to infinity*. (An example is **not** sufficient for full credit.)

d. (2 points) Write the definition of the *cross product* of two vectors. (An example is **not** sufficient for full credit.)

e. (2 points) Let $f = f(x)$. Write the definition of the *Maclaurin series* for f. (An example is **not** sufficient for full credit.)

2. (10 points) Short Answers

a. (5 points) State the *Integral Test* for infinite series.

b. (5 points) State the *nth-Term test* for infinite series.

3. (10 points) Determine whether the following sequences converge or diverge. If a sequence converges, find its limit.

a. (3 points) $a_n = \ln(n+1) - \ln(n)$.

b. (3 points) $b_n = n2^{-\ln(n)}$.

c. (4 points) $a_n = \dfrac{n!}{(-3)^n}$.

4. (10 points) Determine whether the following series converge conditionally, converge absolutely, or diverge.

a. (5 points) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$.

b. (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{0.999999}}$.

5. (10 points) Determine the values of x for which the following series converges conditionally, converges absolutely, or diverges. What are the center, radius, and interval of convergence?

$$\sum_{n=2}^{\infty} n^n x^n.$$

6. (10 points) Compute (no shortcuts!!) the *Taylor series* centered at 1 for the function

$$f(x) = \sqrt{x}.$$

7. (10 points) Using what you know about familiar Taylor series, write a power series for $f(x) = e^{-x^2}$. Use the first 3 non-zero terms to estimate $\int_0^1 e^{-x^2}\, dx$.
(Hint: you should be familiar with the series for e^x.)

8. (10 points) Let P be the point $(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}, -1)$, and Q be the point $(-\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{4}, -1)$.

a. (3 points) Find the component form for $\overrightarrow{PQ}$.

b. (3 points) Find the magnitude $|\overrightarrow{PQ}|$.

c. (4 points) Find the unit vector in the direction of $\overrightarrow{PQ}$.

9. (10 points) Let $\vec{u} = 2\vec{\imath} + 3\vec{\jmath} - \vec{k}$, and $\vec{v} = 3\vec{\imath} - 2\vec{\jmath} + 20\vec{k}$.

a. (3 points) Find $\vec{u} \cdot \vec{v}$.

b. (3 points) Find $\vec{u} \times \vec{v}$.

c. (4 points) Find $\vec{u} \cdot (\vec{u} \times \vec{v})$.

10. (10 points) Find parametric equations for the line which passes through $(2, 4, 5)$ and is perpendicular to the plane $3x + 7y - 5z = 21$.

Bonus. (10 points) Let p_n denote the nth prime: $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$, $p_5 = 11$, etc. Determine whether the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{1}{p_n^2}$$