# NOTES FOR MTH 234
# INSTRUCTORS; SPRING, 2007

The goal of this course is the theorems of Green, Stokes and Gauss.

1 The formula for the distance between a line and a point works only in dimensions 2 and 3. I suggest using instead the formula $d = \left( |\overrightarrow{PS}|^2 - |\operatorname{proj}_{\mathbf{v}} \overrightarrow{PS}|^2 \right)^{\frac{1}{2}}$.

2 The text doesn't emphasize that absolute values are essential in the formula for the angle between two planes.

3 The concepts of limit, continuity and integration for functions of several variable should be presented as extensions of the corresponding one variable notions. For limits and continuity, this is easy to do because the definitions aren't too different, but it would be well to establish the approach for the remainder of the material to point out that continuity is a simple extension of the one variable definition. In all cases the one variable concept should be reviewed first and then the several variable notion defined. Do this first for limits and continuity. Supplemental material has been prepared for this purpose. It may be freely distributed to your students.

4 Continuing the approach suggested above, the text's approach to differentiation is backwards. Introduce the definition of differentiability first and use it to lead naturally to partial derivatives. The supplemental material mentioned above contains this approach to differentiation.

5 The presentation of the Chain Rule in Section 14.4, page 996 can be unified. Supplemental materials have been written on the subject. The basic idea in all forms of the Chair Rule is that the composition of two differentiable functions is differentiable. Students have very little trouble grasping the formula for computing partial derivatives, but few realize that the hypotheses of differentiability is necessary for the validity of the formula. Supplemental material on the Chain Rule is available and may be freely distributed to the students.

6 Restate the definition of directional derivative in Section 14.5 page 1007 first to include functions of any number of variables; not just two and, more importantly, make the limit in the definition as $s \to 0^+$. Without this change the function $f(\mathbf{x}) = |\mathbf{x}|$ doesn't have any directional derivatives at the origin. It should have every directional derivative $=1$ at the origin in any direction.

7 The hypotheses of Theorem 9 in Section 14.5, page 1009 is overstated. The correct hypothesis, "If $f$ is differentiable at $P_0$".

8 The first assertion in Section 14.7, page 1027 should be stated as the Extreme Value Theorem for Functions of Several Variables.

9 Begin Chapter 15 with a review of $\int_a^b f(x)\,dx$ and point out the aspects that will be common to any concept of integration. Consult the supplemental material written for this purpose.

10 There's a serious problem with the equations relating polar and cartesian coordinates appearing on page 1115. The first two equations uniquely determine $x$ and $y$ given $r$ and $\theta$, but the last two don't determine $r$ and $\theta$ given $x$ and $y$ if they are carelessly used. For example for $P = (-1, -1)$. Clearly $r^2 = 2$ and $\tan\theta = 1$; so $r = \sqrt{2}$, $\theta = \frac{\pi}{4}$ is a solution, but $(\sqrt{2}, \frac{\pi}{4})$ isn't a set of polar coordinates for $P$. I suggest listing only the first three equations. Then explaining that the first two uniquely determine $x$ and $y$ given $r$ and $\theta$. Recall that given $x$ and $y$ using $x^2 + y^2 = r^2$ gives **two** possible choices for $r$. For each selection, there is a unique $\theta \in [0, 2\pi)$ satisfying the first two equations. The other polar coordinates for $P$ can be determined from these two selections.

11 The assertion 3. in the definition of spherical coordinates on page 1119 is misleading. The only choices possible for $\theta$ in spherical coordinates are those corresponding to $r \geq 0$. The reason is that in spherical coordinates, $r = \rho \sin\phi \geq 0$.

12 Both cylindrical and spherical coordinates are introduced in Section 15.6, but there are no problems in the text to acquaint the students with theses systems. Probably none is required for cylindrical, but for spherical some are needed and are provided in the Supplemental Exercises for Section 15.6

13 The definition of work on page 1152 should be $W = \int_C \mathbf{F} \cdot \mathbf{T}\,ds$. Similar adjustments need to be made in the list on page 1154. The same change needs to be made in the definition of flow on page 1155.