Math 133, Sections 6-10 Solution Quiz 12 December 4, 2018

1) Consider the curve with parametric equation

$$x = 1 + \frac{1}{t^2}, y = 1 - \frac{1}{t^3}, 0 \le t \le 10.$$

Determine the equation of the tangent line to this curve at $(\frac{5}{4}, \frac{7}{8})$.

$$(x,y) = \left(\frac{5}{4}, \frac{7}{8}\right) \Rightarrow 1 + \frac{1}{t^2} = \frac{5}{4} \text{ and } 1 - \frac{1}{t^3} = \frac{7}{8}$$

$$\Rightarrow t^2 = 4 \text{ and } t^3 = 8$$

$$\Rightarrow t = \pm 2 \text{ and } t = 2$$

$$\Rightarrow \boxed{t = 2}$$

$$x'(t) = (1 + t^{-2})' = -2t^{-3} = \frac{-2}{t^3}$$

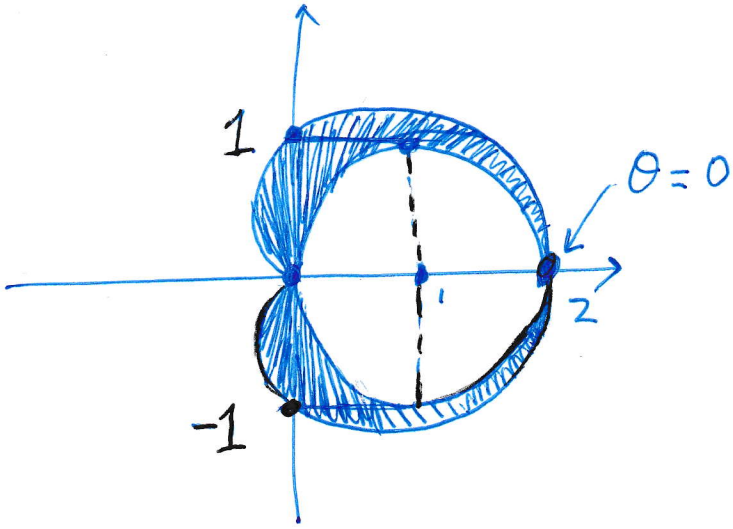
$$y'(t) = (1 - t^{-3})' = 3t^{-4} = \frac{3}{t^4}.$$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{3/t^4}{-2/t^3} = \frac{3}{t^4} \cdot \frac{t^3}{-2} = -\frac{3}{2t}.$$

$$\left.\frac{dy}{dx}\right|_{t=2} = \frac{-3}{4}.$$

Eqn of tangent line: $\boxed{y - \frac{7}{8} = -\frac{3}{4}\left(x - \frac{5}{4}\right)}$

2) Set up BUT DO NOT EVALUATE the integral to determine the area outside the circle $r = 2\cos\theta$ and inside the cardioid $r = 1 + \cos\theta$. You should draw the polar graphs first.



Cardioid

θ	r
0	2
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	1

Circle

θ	r
0	2
$\frac{\pi}{2}$	0
π	-2

Area = Area of cardioid − Area of circle

Symmetry

$$= 2 \cdot \frac{1}{2}\int_0^{\pi} (1+\cos\theta)^2\, d\theta - \pi \cdot (1)^2$$

$$= \int_0^{\pi} (1+\cos\theta)^2\, d\theta - \pi.$$

or $$A = \underbrace{\int_0^{2\pi} \frac{1}{2}(1+\cos\theta)^2\, d\theta}_{\text{Area of Cardioid}} - \underbrace{\int_0^{\pi} \frac{1}{2}(2\cos\theta)^2\, d\theta}_{\text{Area of Circle}}.$$