

Solution

Quiz #10

1. (2 points) Find a parametrization of the line segment starting at the point $(-2, 3)$ and ending at the point $(3, 1)$ using linear functions $x(t)$ and $y(t)$ with $0 \leq t \leq 1$.

$$\begin{aligned} x(t) &= at + b & \left| \begin{array}{l} x(0) = b = -2 \\ x(1) = a - 2 = 3 \Rightarrow a = 5 \end{array} \right. \\ y(t) &= ct + d & \left| \begin{array}{l} y(0) = d = 3 \\ y(1) = c + 3 = 1 \Rightarrow c = -2 \end{array} \right. \end{aligned}$$

$$\begin{aligned} x(t) &= 5t - 2 \\ y(t) &= -2t + 3 \end{aligned}$$

2. (3 points) Consider the following pairs of polar coordinates (r, θ) . Do they describe the same point in the plane? Write 'T' if they do and 'F' if they do not.

$$\left(16, \frac{32\pi}{3}\right), \left(-16, -\frac{\pi}{3}\right) \quad \text{T}$$

$$\left(2, \frac{\pi}{3}\right), \left(-2, -\frac{\pi}{3}\right) \quad \text{F}$$

$$\left(16, \frac{32\pi}{3}\right), \left(-16, \frac{\pi}{3}\right) \quad \text{F}$$

$$(2, 5\pi), (-2, 5\pi) \quad \text{F}$$

$$\left(1, \frac{21\pi}{4}\right), \left(-1, \frac{\pi}{4}\right) \quad \text{T}$$

$$(0, \pi), (0, \frac{\pi}{2}) \quad \text{T}$$

3. (5 points) Consider the equation $r = 4 \sin \theta$ in polar coordinates. What is the equivalent equation in cartesian coordinates? What kind of curve is given by this equation? Describe it in detail or sketch it.

$$r = 4 \sin \theta \Rightarrow r^2 = 4 r \sin \theta \Rightarrow x^2 + y^2 = 4y \Rightarrow x^2 + y^2 - 4y = 0.$$

$$x^2 + (y-2)^2 - 4 = 0 \Rightarrow x^2 + (y-2)^2 = 2^2 \quad \text{equation of the}$$

circle centered at $(x, y) = (0, 2)$ of radius 2.

