

Quiz #10

1. (2 points) Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - 2x}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{-1 - 2x + 1 + 2x + \frac{4x^2}{2} + \frac{8x^3}{6} + \dots}{x^2} = \lim_{x \rightarrow 0} \frac{2x^2 + \frac{4}{3}x^3 + \dots}{x^2}$$

$$= \lim_{x \rightarrow 0} \left(2 + \frac{4}{3}x + \dots \right) = \boxed{2}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

$$e^{2x} = 1 + 2x + \frac{4x^2}{2} + \frac{8x^3}{6} + \dots$$

2. (5 points) Find the second degree Taylor polynomial of $f(x) = \tan x$ at $x = \frac{\pi}{4}$.

$$f(x) = \tan x, f'(x) = \sec^2 x, f''(x) = 2 \sec x \cdot \sec x \cdot \tan x = 2 \sec^2 x \tan x.$$

$$f\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1, \quad f'\left(\frac{\pi}{4}\right) = (\sec\left(\frac{\pi}{4}\right))^2 = (\sqrt{2})^2 = 2, \quad f''\left(\frac{\pi}{4}\right) = 2 \cdot 2 \cdot 1 = 4.$$

$$\begin{aligned} T_2(x) &= f\left(\frac{\pi}{4}\right) + \frac{f'\left(\frac{\pi}{4}\right)}{1!} \cdot (x - \frac{\pi}{4}) + \frac{f''\left(\frac{\pi}{4}\right)}{2!} \cdot (x - \frac{\pi}{4})^2 \\ &= 1 + 2(x - \frac{\pi}{4}) + \frac{4}{2} (x - \frac{\pi}{4})^2 = \boxed{1 + 2(x - \frac{\pi}{4}) + 2(x - \frac{\pi}{4})^2} \end{aligned}$$

3. (3 points) Sketch the curve

$$x(t) = \sin t, \quad y(t) = \cos t, \quad \frac{\pi}{2} \leq t \leq \frac{3\pi}{2}.$$

Also indicate in which direction the curve is traversed.

$$x^2(t) + y^2(t) = \sin^2(t) + \cos^2(t) = 1$$

$x^2 + y^2 = 1$ is the equation of the circle of radius 1 centered at $(0, 0)$.

t	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	Plot
x	1	0	-1	
y	0	-1	0	

