Name: Solution **Section:** 11

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (5 points) Find the interval of convergence for the following series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n (2x)^n}{n!}$$

By the ratio test, we want $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(-1)^{n+1}(2x)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-1)^n (2x)^n}\right|$$

$$= \lim_{n\to\infty} \left|\frac{2x}{n+1}\right| = \frac{|2x|}{\infty} = 0 < 1 \quad \text{for all } x.$$

Therefore, the Radius of Convergence for the Series above is $\boxed{R = \infty}$

2. (5 points) Consider the function $f(x) = \dfrac{8}{1+5x}$.

(a) Find the power series representation for f in sigma notation.

We know the geometric series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, $|x| < 1$.

Thus, $\frac{8}{1+5x} = 8 \cdot \frac{1}{1-(-5x)} = 8 \cdot \sum_{n=0}^{\infty} (-5x)^n = \boxed{\sum_{n=0}^{\infty} 8 \cdot (-5)^n x^n}$

(b) Give its radius of convergence.

We need the ratio of the geom. Series to be less than 1 in Absolute value: $|-5x| < 1 \iff |5x| < 1 \iff |x| < \frac{1}{5}$.

Therefore, the radius of convergence is $\boxed{R = \frac{1}{5}}$.