

Solution.

Math 133, Quiz #9

1. Find the interval of convergence and the convergence radius of the following power series:

(a)

This is a geometric Series, with ratio $r = x$. It converges when $|r| = |x| < 1$, that is, for $-1 < x < 1$. Therefore, the interval of convergence is ($-1, 1$), and the radius of convergence is $R = 1$

(b)

$$\sum_{n=0}^{\infty} x^n \quad (3 \text{ points})$$

$$\text{Let } a_n = \frac{(4x-5)^{2n+1}}{n^{3/2}} \quad (4 \text{ points})$$

We want $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$ by the ratio test for convergence.

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(4x-5)^{2n+3}}{(n+1)^{3/2}} \cdot \frac{n^{3/2}}{(4x-5)^{2n+1}} \right| = \lim_{n \rightarrow \infty} \left| (4x-5)^2 \cdot \left(\frac{n}{n+1} \right)^{3/2} \right|$$

$$= |4x-5|^2 \cdot \left(\lim_{n \rightarrow \infty} \frac{n}{n+1} \right)^{3/2} = |4x-5|^2 \cdot 1^{3/2} = \boxed{(4x-5)^2}$$

$$\text{Now, } (4x-5)^2 < 1 \iff -1 < 4x-5 < 1 \iff 4 < 4x < 6$$

$$\Leftrightarrow 1 < x < \frac{3}{2}$$

The interval of convergence is ($1, \frac{3}{2}$), and the radius of convergence is $R = \frac{1}{4}$.

2. Find the Mac Laurin series of the function $f(x) = \ln(1 + x^2)$. (Hint: Use geometric series)(3 points).

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \Rightarrow \frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$\ln(1+x) = \int \frac{1}{1+x} dx = \sum_{n=0}^{\infty} \int (-1)^n x^n dx = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} + C$$

for $x=0$ $\ln(1+0)=0 = \sum_{n=0}^{\infty} (0) + C \Rightarrow C=0$

So, $\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$. Replacing x by x^2 , we finally get

$$\begin{aligned} \ln(1+x^2) &= \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{n+1}}{n+1} \\ &= \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+2}}{n+1}}. \end{aligned}$$