Name: Solution Section: 11

If you have a question raise your hand and I will come to you.

Consider each infinite series below. Determine whether the series converges or diverges. If possible, find the sum of the series. You must show all of your work– show your reasons for deciding to use a certain test and support your conclusions.

1. (5 points) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ Let $a_n = \frac{\cos n}{n^2}$; Then $|a_n| = \frac{|\cos(n)|}{n^2}$.

Let $b_n = \frac{1}{n^2}$. Then $|a_n| > 0$ and $b_n > 0$ for all $n$, and $\sum b_n$ converges by p-Series, $p = 2 > 1$. Now,

$|a_n| = \frac{|\cos(n)|}{n^2} < \frac{1}{n^2} = b_n$ for all $n \geq 1$. Therefore, $\sum |a_n|$ converges by the D.C.T, So $\sum a_n$ converges absolutely, and thus converges.

2. (5 points) $\sum_{n=1}^{\infty} \frac{(-40)^n}{n!}$ Let $a_n = \frac{(-40)^n}{n!}$. Then

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(-40)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-40)^n} \right| = 40 \cdot \lim_{n\to\infty} \left| \frac{n!}{(n+1)!} \right|$$

$$= 40 \cdot \lim_{n\to\infty} \left| \frac{1}{n+1} \right| = 40 \cdot 0 = 0 < 1.$$

Therefore, by the ratio test, $\sum a_n$ **Converges absolutely, and therefore converges.**