

Solution1) Let  $\{a_n\}$  be the sequence given by

$$a_n = \frac{\ln(1 + \frac{2}{n})}{\sin \frac{1}{n}}$$

Determine if it is convergent.

$$\lim_{n \rightarrow \infty} \frac{\ln(1 + \frac{2}{n})}{\sin(\frac{1}{n})} \quad (\text{form } \frac{\ln(1)}{\sin(0)} = \frac{0}{0}; \text{ indeterminate} \rightarrow \underline{\underline{\text{L'Hop Rule}}})$$

$$= \lim_{n \rightarrow \infty} \frac{\frac{1}{1 + \frac{2}{n}} \cdot (-\frac{2}{n^2})}{\cos(\frac{1}{n}) \left(-\frac{1}{n^2}\right)} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{1}{1 + \frac{2}{n}}}{\cos(\frac{1}{n})} = \frac{2}{\cos(0)} = 2$$

Thus  $\{a_n\}$  converges to 2.

2) Determine the open interval of values of  $x$  for which the series

$$\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} (-2)^n x^n$$

is convergent?

$$\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} (-2x)^n, \text{ a geometric Series w/ ratio } r = -2x. \text{ It converges when } |r| = |-2x| < 1$$

$$\text{Now } |-2x| < 1 \iff -1 < -2x < 1 \iff -\frac{1}{2} < x < \frac{1}{2}$$

The open interval of values for which  $\sum_{n=0}^{\infty} (-2x)^n$  converges is  $(-\frac{1}{2}, \frac{1}{2})$

3) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2}{n^2 + 1}$$

is convergent USING THE INTEGRAL TEST.

Let  $f(x) = \frac{2}{x^2 + 1}$  for  $x$  in  $[1, \infty)$ . Then  $f$  is continuous and positive ( $n^2 + 1 \geq 1$ ) on  $[1, \infty)$ , and decreasing since the denominator increases with  $x$ . Thus we may use the integral test.

$$\begin{aligned}\int_1^{\infty} \frac{2}{x^2 + 1} dx &= 2 \cdot \lim_{t \rightarrow \infty} \int_1^t \frac{1}{x^2 + 1} dx \\ &= 2 \cdot \lim_{t \rightarrow \infty} \arctan(x) \Big|_1^t \\ &= 2 \lim_{t \rightarrow \infty} (\tan^{-1}(t) - \tan^{-1}(1)) \\ &= 2 \left[ \frac{\pi}{2} - \frac{\pi}{4} \right] = 2 \frac{\pi}{4} = \boxed{\frac{\pi}{2}}\end{aligned}$$

Since  $\int_1^{\infty} f(x) dx$  converges, then  $\sum_{n=1}^{\infty} \frac{2}{n^2 + 1}$  converges as well.